WEATHER-RADAR ATTENUATION ESTIMATES FROM RAINGAUGE STATISTICS

by

P. M. Hamilton and J. S. Marshall

STORMY WEATHER GROUP
MACDONALD PHYSICS LABORATORY
McGILL UNIVERSITY

SCIENTIFIC REPORT MW-32

under

Contract No. AF-19 (604)-2065

JANUARY 1961

Prepared for
GEOPHYSICS RESEARCH DIRECTORATE
AIR FORCE CAMBRIDGE RESEARCH LABORATORIES
AIR FORCE RESEARCH DIVISION
UNITED STATES AIR FORCE
BEDFORD, MASSACHUSETTS

SCIENTIFIC REPORT MW-32

WEATHER-RADAR ATTENUATION ESTIMATES FROM RAINGAUGE STATISTICS

ABSTRACT

The attenuation of radar weather signals by intervening precipitation is difficult to estimate from the radar records themselves. Rainfall rates observed at a point in the path of a storm approximate those along a section through the storm; this is the basis in estimating attenuation frequencies at wavelengths 3 cm and 5.7 cm for a summer's storms at Montreal. Statistically, the amount of attenuation along a radar path varies greatly with the intensity of the target rain: Attenuation increases with target intensity. This is mostly attributable to attenuation by rain very close to the target rain, and in the same shower. As a result, areas of intense rain are much reduced and distorted, although the maximum intensity values of showers come closer to the true maxima than had been expected (half the maxima were down by 4 db and one in ten by 11 db). Although attenuation at 5.7 cm is less troublesome than at 3 cm, truly quantitative operation demands 10 cm.

Requests for additional copies by Agencies of the Department of Defense, their contractors, and other Government agencies should be directed to the:

ARMED SERVICES TECHNICAL INFORMATION AGENCY
ARLINGTON HALL STATION
ARLINGTON 12, VIRGINIA

Department of Defense contractors must be established for ASTIA services or have their 'need-to-know' certified by the cognizant military agency of their project or contract.

All other persons and organizations should apply to the:

U.S. DEPARTMENT OF COMMERCE
OFFICE OF TECHNICAL SERVICES
WASHINGTON 25, D.C.

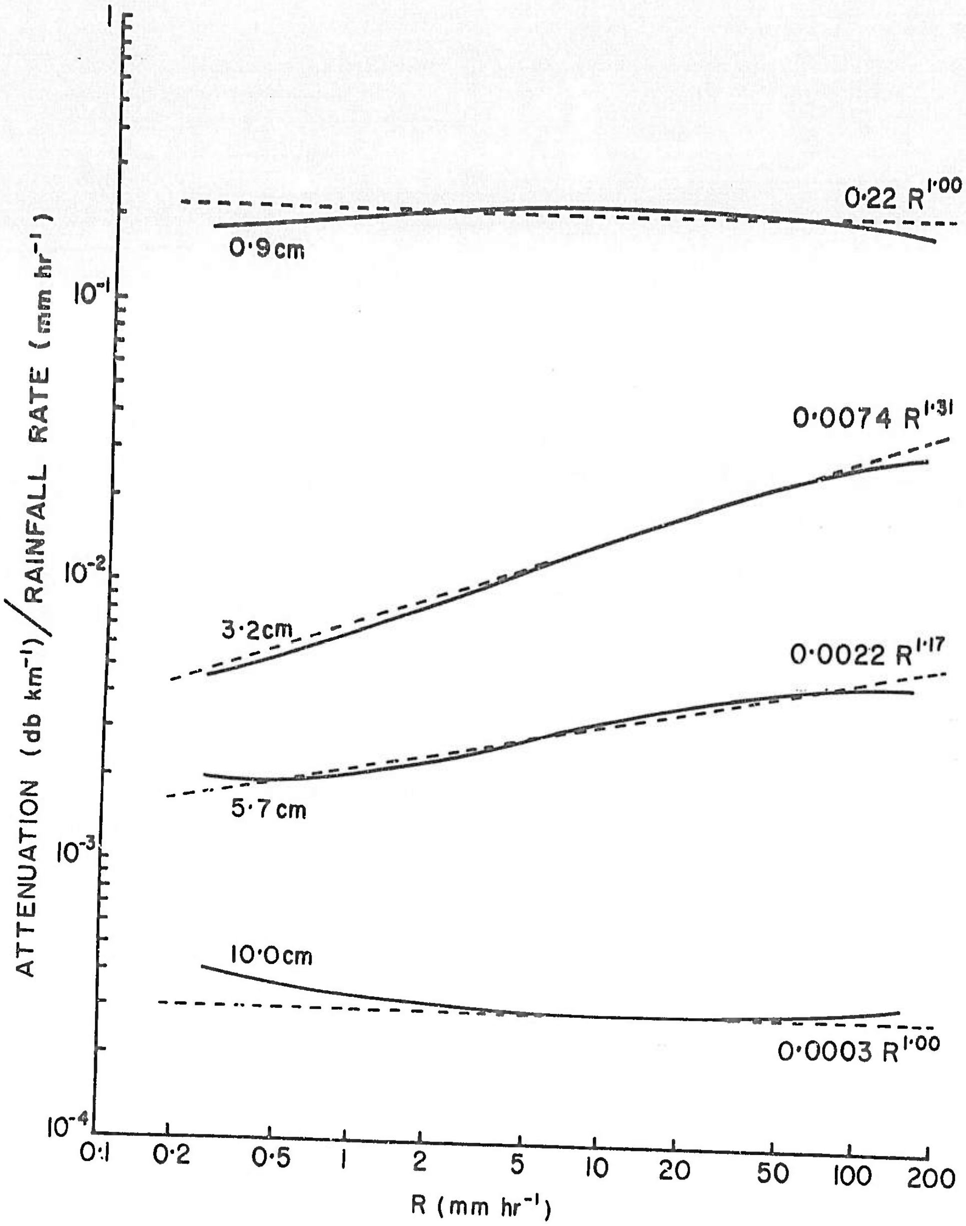
SCIENTIFIC REPORT MW-32

WEATHER-RADAR ATTENUATION ESTIMATES FROM RAINGAUGE STATISTICS

TABLE OF CONTENTS

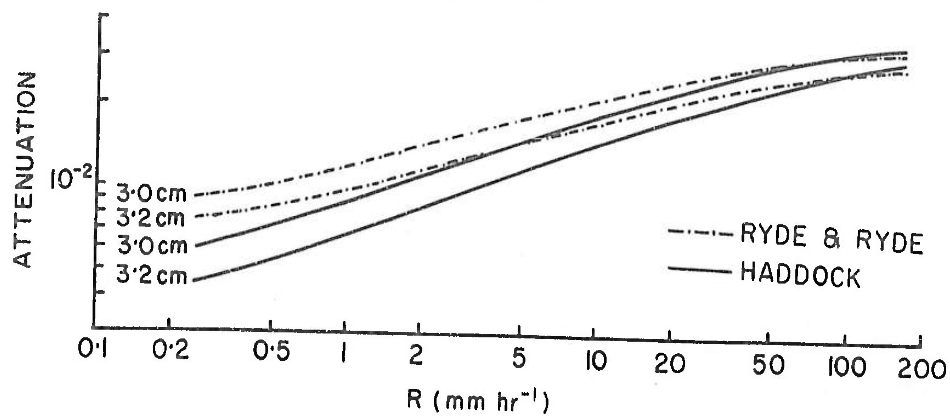
ACKNOWLEDGEMENT OF SUPPORT FROM SOURCES OUTSIDE THE CONTRACT

Part of the work described in this report was contained in Hamilton's M.Sc. thesis, which was submitted in May 1960. During his M.Sc. work Hamilton held an Assistantship from the National Research Council of Canada. The rain-gauge data came from tipping-bucket gauges, provided and maintained by the Meteorological Service of Canada.



ATTENUATION (db km⁻¹)/RAINFALL RATE (mm hr⁻¹)
0·22 R^1·00
0·9 cm
0·0074 R^1·31
3·2 cm
0·0022 R^1·17
5·7 cm
10·0 cm
0·0003 R^1·00
R (mm hr⁻¹)

FIG. 1a.



ATTENUATION
3·0 cm
3·2 cm
3·0 cm
3·2 cm
RYDE & RYDE
HADDOCK
R (mm hr⁻¹)

FIG. 1b.

1. QUANTITATIVE RADAR OPERATION AND RAINGAUGE STATISTICS

Radar circuits and display techniques of wide dynamic range have been developed by the Stormy Weather Group (Legg, 1960; Marshall and Gunn, 1960) so that radar pictures would reveal (by shades of grey) the intensity of precipitation at every point on a map. Use of these circuits and techniques with a CPS-9 radar, wavelength 3.2 cm, indicated that at that wavelength the quantitative information that it had been hoped to gain was seriously impaired by the attenuation of the 3.2-cm radiation by rainfall.

Determination of the amount of attenuation requires knowledge of

(1) The relation between attenuation and rainfall rate (Fig. 1), and

(2) The amount of precipitation intervening between the radar and the target rainfall.

The experience with the modified CPS-9 mentioned above served as a reminder of the need for statistical or climatological information about attenuation and, toward that end, about item (2) above.

FIG. 1a. Attenuation/rainfall-rate versus rainfall-rate for various wavelengths. The values in the figure are for one-way travel: For use in radar it is necessary to recognise that there is attenuation both in the path from the radar to the target and in the return path. The solid curves were obtained as follows: The one for 0.9 cm was drawn through Haddock's (1948) data. The curve for 3.2 cm involved extrapolating slightly beyond the limit of Haddock's data at 3.0 cm. The 10.0-cm curve is drawn through the data of Ryde and Ryde (1945). The curve at 5.7 cm is interpolated between the available data at 3 and 10 cm. The broken straight lines are approximations. Beside each of these lines the attenuation per kilometre (one-way) in decibels is given as a function of rainfall rate in mm hr^{-1}. (These functions come from Table 5 of Gunn and East (1954); all the solid lines except the 3.2-cm one come from their Fig. 5).

FIG. 1b. This figure is a detail and expansion of Fig. 1a. The solid curve at 3.0 cm is drawn through Haddock's (1948) data (it is this, rather than a 3.2-cm curve, that appears in Gunn and East's (1954) Fig. 5); the solid curve at 3.2 cm, from an extrapolation of Haddock's data, has already appeared in Fig. 1a. The broken curves are both drawn through data given in Table IX of Ryde and Ryde (1945).

In the work described in this report we considered wavelength 3.0 cm, rather than 3.2 cm, taking our data from the 3.0-cm curve of Gunn and East's Fig. 5. The attenuation at 3.0 cm, in db, is about 20% greater than at 3.2 cm.

The distribution of rain with rate of rainfall is shown in Fig. 2. The data are for the season May to September 1959, from three tipping-bucket rain-gauges (0.01 inch per tip) located relative to the radar at Montreal Airport as follows:

L'Assomption	29 mi	30°
Farnham	39 mi	107°
Huntingdon	33 mi	220°

The gauges averaged 150 hours and 347 mm of rain, per gauge, in the season.

It is reasonable to assume that Fig. 2 is representative of any point on the weather radar map. The distribution of "duration" for these gauges then serves as a distribution of "area" on the map or radar display: Area of countryside over which it is raining, area of "echo" or radar return on the display. This should be true for the season, and might also be expected to hold, as a first approximation, for any instant at which the picture contains a considerable

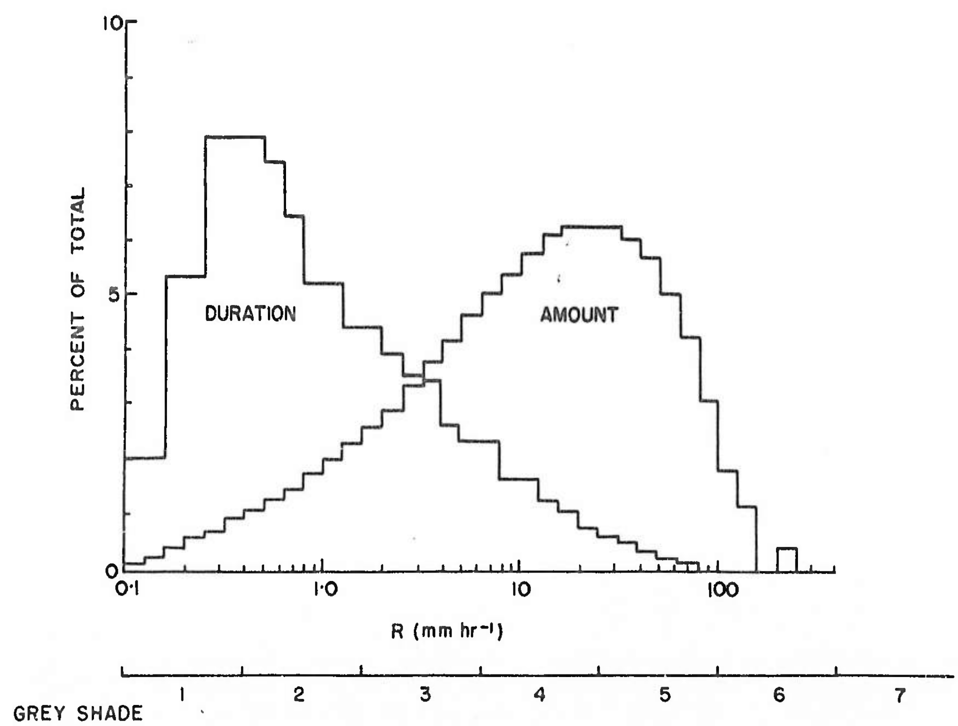


FIG. 2. Distribution of duration and amount of rain with rate of rainfall.

amount of typical weather. The distribution of "amount", i.e. of amount of rainfall, should also be applicable to the amount per unit area at a point on the map, or the amount over any considerable portion of the countryside, determined from a (non-attenuating) radar.

It is relevant to note the sort of boundary that exists, in Fig. 2, at 3 mm hr^{-1}: 80% of the amount of rainfall occurs at rates above this value, while 80% of the duration is at rates below. Thus, in the season May to September, 80% of the echo-area of a radar screen reports rain at rates less than 3 mm hr^{-1}, yet contributes only 20% of the amount of rainfall. Most of the significant information provided by the radar regarding severe storms is contained in the high-intensity tail of the "duration" distribution, and it is important that this most significant minority be distinguishable from the less significant majority of echo coming from light rain.

The circuits of the radar at Montreal were designed to respond in discrete steps (denoted on the display by successively lighter shades of grey) at seven points, with factors four in rainfall rate between successive steps, as shown along the bottom of Fig. 2. The positions of the steps varied considerably in the course of the season, but the factors of four between steps were held constant. The high-intensity steps extended beyond the most intense rain recorded by the gauges, in the hope of observing particularly strong signals such as Donaldson (1958) has reported, particularly at about 20,000 ft, from severe storms. In practice, and neglecting attenuation, the durations of shades 3 and beyond, at the points on the pictures corresponding to the raingauge locations, were small in comparison with durations of corresponding rates of rainfall recorded by the gauges. It was recognised that this scarcity of strong signals, relative to weak ones, could be explained by attenuation, if attenuation tends to increase with increasing rate of target rainfall. (Appendix I)

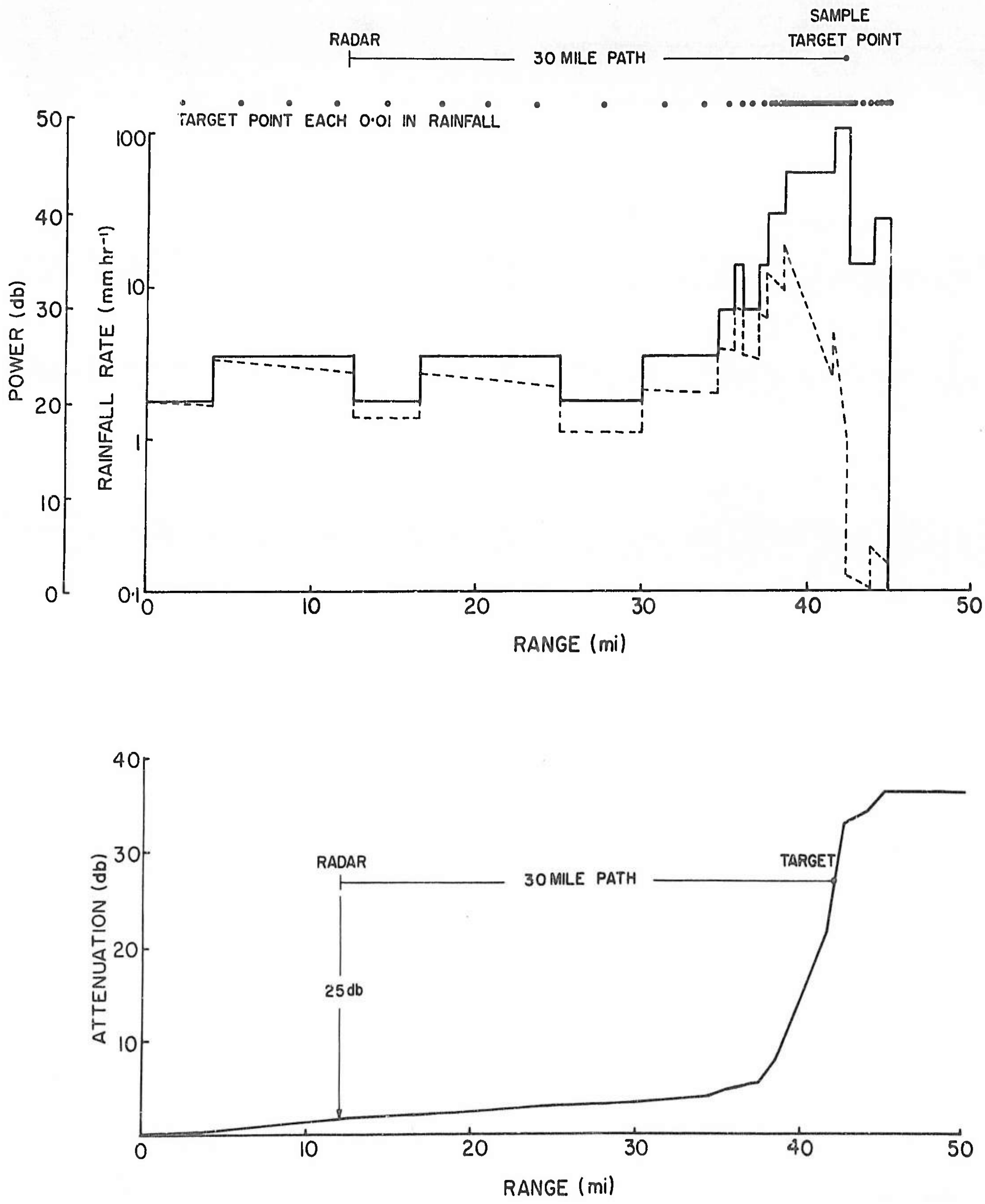


FIG. 3. Above: Typical synthetic storm (solid line), and the storm as viewed on 3.0-cm radar (broken line).
Below: 3.0-cm attenuation in the storm as a function of range.

2. ATTENUATION INFORMATION FROM RAINGAUGE DATA

The effect of attenuation in a typical case is demonstrated in Fig. 3. The solid line has been drawn to represent rainfall rate as a function of distance. The broken line, correspondingly, represents apparent rainfall rate, as it would be indicated by a 3.0-cm radar well-calibrated except for complete neglect of attenuation. Conversely, application of the attenuation relation for 3.0 cm (Fig. 1b) would correct it to the solid line. It must be noted with regard to this converse that a slight under-estimate of radar performance will drive the corrected curve to absurdly high rainfall rates.

For statistical purposes, how can curves such as the solid line of Fig. 3 be obtained? Records from 3-cm radar would have to be corrected for attenuation and this was not an attractive prospect.* Records from 10-cm radar, of the requisite wide dynamic range, are not thought to be available in sufficient quantity or with sufficient continuity. The solid line of Fig. 3 was actually obtained from raingauge records, with an assist, so to speak, from radar. Rate of rainfall was plotted against time for the gauge at L'Assomption. The scale of the abscissa was then converted to a distance scale by introducing the speed of horizontal motion of the precipitation pattern. This speed was obtained from radar sequences at the appropriate time, or at worst on the appropriate day. Development of the precipitation pattern (apart from translation) was neglected. This neglect should be less serious for the acquisition of statistics than it is in short-range forecasting, and forecasting by simple extrapolation has been found useful for periods up to one hour.

---

* Hitschfeld and Bordan (1954) concluded that correcting for attenuation involved remarkable difficulties, but that these difficulties might conceivably be overcome. New attempts in this laboratory this year to correct for attenuation justify hope that practicable techniques will be achieved.

The speeds that were involved are shown in Fig. 4; directions were largely from bearings between 240° and 300°. An objection to this approach is that precipitation may tend to be more, or less, extensive in the direction of its motion than in the direction at right angles. There is some evidence against this objection in the comparison that was made of 3.2-cm radar with raingauges. The shortcomings of the radar were much the same at the three gauge locations, although the directions in which they lay were quite different: 30°, 107°, 220°.

Fig. 3 is representative of graphs covering, for the gauge at L'Assomption, all rain in the period May to September 1959. To compile these graphs, the range of rainfall rates from 0.31 to 320 mm hr^{-1} was divided into ten intervals, equal on a log scale and so each separated from the next by a factor two. All rainfall rates within an interval were assigned the value of the geometric mean of that interval. One small inadequacy of the tipping-bucket gauge may be noted: In each storm, up to 0.02 inch of rain was unaccounted for, because there was no record of the exact onset or end of the storm. There could be more serious errors in recording high rainfall rates, but a laboratory test

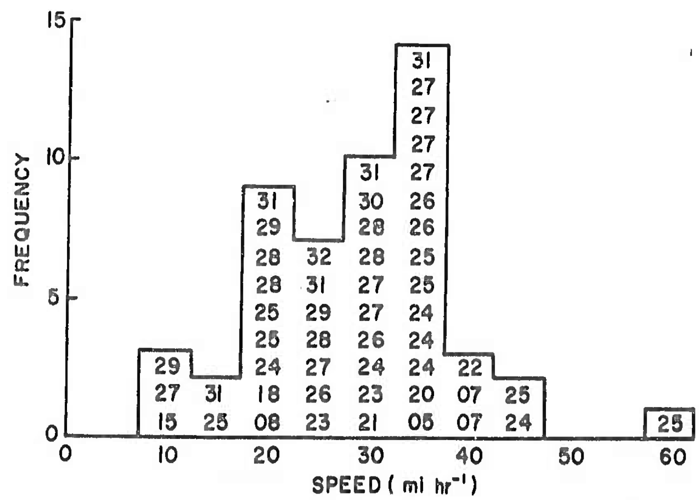


FIG. 4. The distribution of storms with radar-determined speed. A storm is at least 120 miles from a neighbour, and yields at least 0.02 inch rain. For each speed, the directions from which storms of that speed moved are shown in tens of degrees. In addition to the 51 storms shown, there were seven storms of unknown speed, each of which was assigned speed 30 mi hr^{-1} for the purposes of the present study.

indicated accurate recording up to 250 mm hr^{-1}, at least, and so beyond rates observed during the season.

In the lower half of Fig. 3, the two-way attenuation at wavelength 3.0 cm has been plotted against range. The attenuation along a path between any two ranges is readily obtained, it being the difference between the values of the attenuation at the two ranges. The primary aim of the study was to investigate the dependence of attenuation, along a 30, or 60, or 120-mile path, on the rainfall rate at one end of the path. The rainfall at this end of the path represented target rainfall under observation by a radar at the other end of the path. It was the comparison of radar and raingauge records that led to the notion that attenuation along a thirty-mile path tended to be higher when the target rain at the end of the path was heavy than when it was light. It is quite obvious, come to think of it: A few miles of heavy rain effects great attenuation (right side of Fig. 3), and there is much more chance of a few miles of heavy rain adjacent to a point at which the rain is heavy than there is of it existing within 30 miles (on a given bearing) of a point at which the rain is light.

3. REDUCTION OF DATA

In treating the graphical data of which Fig. 3 is a sample, targets were selected for each 0.01 inch of rainfall. The reason for each 0.01 inch of rain rather than each minute of time was that this technique yields data fairly evenly distributed over rainfall rates between 2 and 80 mm hr^{-1}, with a few at higher and lower rates. With one-minute intervals there would have been very few data at the interesting high rates and many data at low rates. This is reflected in Fig. 2, which shows both amount and duration as functions of rainfall rate.

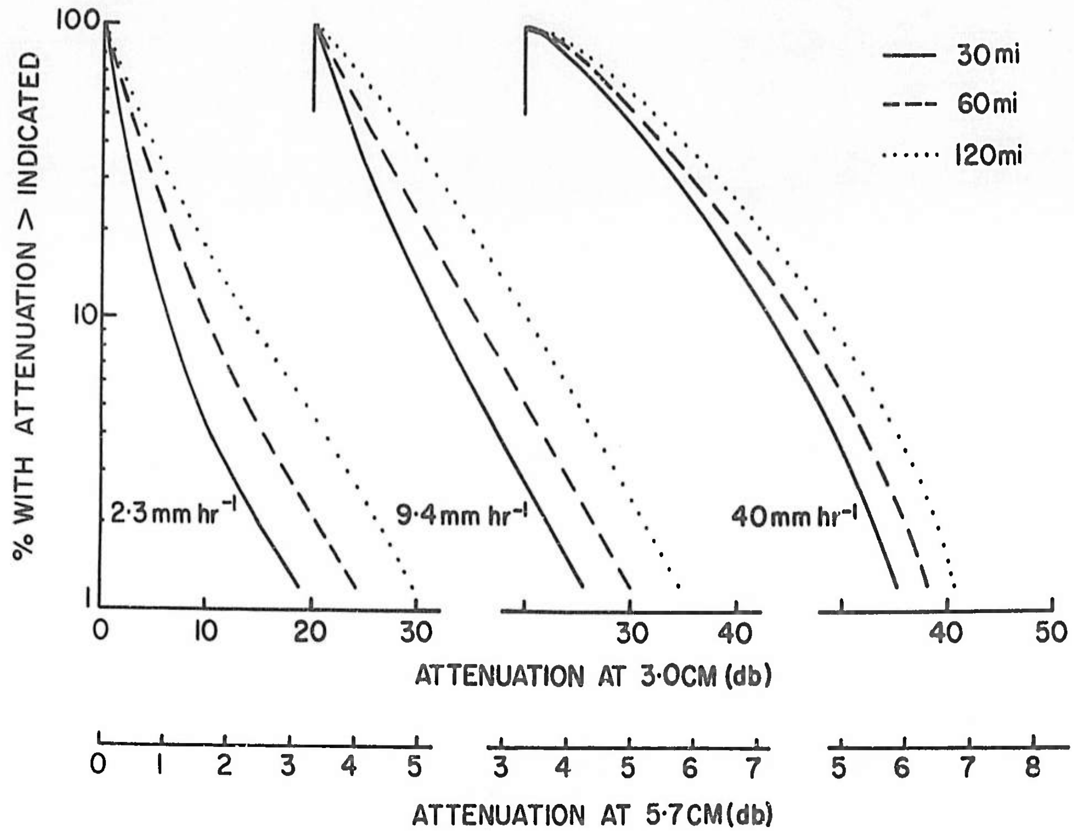


FIG. 5. (Above) Dependence of attenuation on target range.

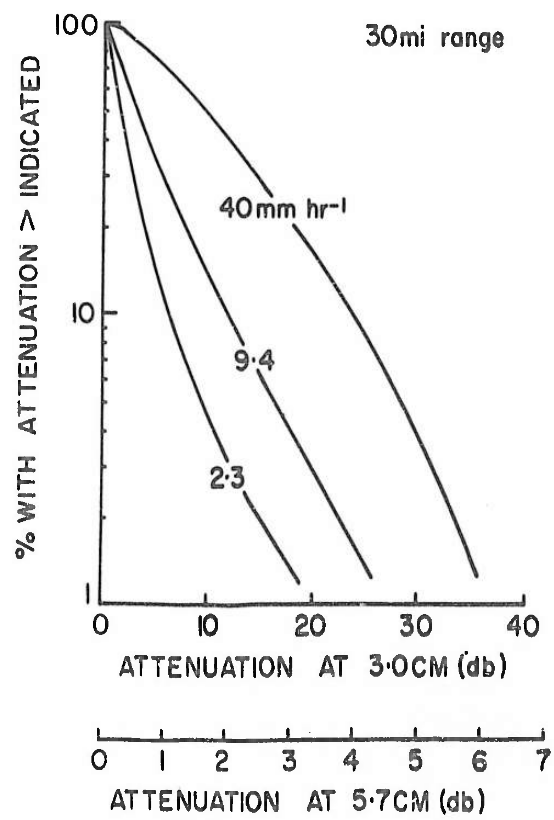


FIG. 6. (Left) Dependence of attenuation on target rainfall rate.

The record of the tipping-bucket gauge indicates the time interval during which each 0.01 inch was accumulated, and a target was taken at mid-point of each of these intervals. For each target the attenuation along paths 30, 60 and 120 miles to weatherward was calculated. Thus, at the very top of Fig. 3, the target has been located at range 42 miles. When the radar was located at range 12 miles, the attenuation over a 30-mile path to the target was 25 db; over paths of 42 miles or greater the attenuation was 27 db. Values of attenuation were measured to 1 db.

As already mentioned the range of rainfall rates was divided into ten intervals and this division was also applied to target rates. Using the technique described above, a tabulation of the frequency of various values of attenuation for each of the ten intervals was made. From this tabulation, cumulative distributions for the intervals were drawn up. These distributions were of the same sort as the loci of Figs. 5 and 6, but more numerous and less certain; the loci of those figures were drawn for wider intervals in the hope of bringing out family characteristics. Finally, a closely-spaced family of twenty curves was interpolated and extrapolated; these are not shown, but led to the tables that are discussed in the next paragraphs. For the extrapolation below 1.25 mm hr^{-1}, there was some suggestion in the sparse data that attenuation becomes independent of target rainfall rate in this low-intensity region, as might be expected from physical argument. There is no evidence of anything similar at high rainfall rates; independence was assumed above 80 mm hr^{-1}, but this probably results in underestimates of attenuation in this high-intensity region.

TABLE I - APPARENT MODIFICATION OF RAINFALL RATE DISTRIBUTION BY ATTENUATION AT 3.0-CM WAVELENGTH OVER 30-MI PATH

RAINFALL RATE (mm hr^{-1})																						RAINFALL		
<.09	.11	.15	.22	.30	.45	.60	.90	1.2	1.8	2.5	3.5	5	7	10	14	20	28	40	56	80	>92	RATE	FREQ	CUM
											1	1	1	1	1	1	1	1	1	1		>92	10	10
							1	1	1	1	1	1	1	1	1	2	2	1	1			80	15	25
1					1	1	1	1	2	3	3	4	4	4	5	5	5	4	1			56	45	70
1					1	1	1	2	3	4	4	6	7	8	8	10	10	4				40	70	140
1				1	1	1	1	3	3	5	6	7	9	13	14	18	7					28	90	230
1				1	1	2	2	4	5	8	11	13	18	25	35	14						20	140	370
1				1	2	2	3	5	8	11	17	25	40	60	25							14	200	570
1			1	1	2	3	4	8	10	20	30	50	100	40								10	270	840
4	1	1	2	2	2	4	6	12	16	30	70	140	60									7	350	1190
8	2	2	2	2	2	3	5	14	30	60	160	90										5	380	1570
4	2	2	3	4	6	10	15	34	60	180	160											3.5	480	2050
4	1	1	2	4	7	13	31	57	210	240												2.5	570	2620
5	1	3	4	7	10	20	40	170	370													1.8	630	3250
7	3	4	7	14	25	50	230	460														1.2	800	4050
7	4	8	15	26	60	240	490															.9	850	4900
10	10	20	30	70	280	580																.6	1000	5900
30	20	30	70	310	640																	.45	1100	7000
50	30	70	310	640																		.30	1100	8100
80	70	310	640																			.22	1100	9200
70	140	290																				.15	500	9700
130	170																					.11	300	10000
																						<.09	0	10000
420	450	740	1090	1080	1040	930	830	770	720	560	460	340	240	150	90	50	25	10	3	1				
10000	9600	9100	8400	7300	6200	5200	4250	3400	2650	1930	1370	910	570	330	180	90	40	15	4	1				

Tables I to IV have been compiled for two wavelengths and two target ranges. Table I is for wavelength 3.0 cm and range 30 miles. The frequencies in the right-hand margin show how a normalized 10,000 hours of rainfall exceeding a rate of 0.09 mm hr^{-1} are distributed with rainfall rate. (The data are from the same source as Fig. 2). Each row shows how the hours at a particular rainfall rate are redistributed by attenuation among the same intervals of apparent rainfall rate. For example, of the 380 hours of rain falling at rate 5 mm hr^{-1}, 90 hours are observed as 5 mm hr^{-1}, 160 hours as 3.5 mm hr^{-1} and so on, with 8 hours attenuated below the assumed threshold rainfall rate of 0.09 mm hr^{-1}. Each column shows how the hours at a particular apparent rainfall rate originate from various actual rainfall rates. The second-from-bottom row contains totals of the hours in each column, and therefore shows how total hours are distributed among the apparent rainfall rates. As mentioned with reference to Fig. 2, "duration" for a raingauge becomes area of echo, or area-

TABLE II - APPARENT MODIFICATION OF RAINFALL RATE DISTRIBUTION BY ATTENUATION AT 5.7-CM WAVELENGTH OVER 30-MI PATH

RAINFALL RATE (mm hr^{-1})																						RAINFALL		
< .09	.11	.15	.22	.30	.45	.60	.90	1.2	1.8	2.5	3.5	5	7	10	14	20	28	40	56	80	> 92	RATE	FREQ	CUM
																			2	6	2	> 92	10	10
																		3	8	4		80	15	25
																1	7	24	13			56	45	70
															1	7	33	29				40	70	140
														2	5	36	47					28	90	230
													1	7	47	85						20	140	370
													6	54	140							14	200	570
												5	65	200								10	270	840
											4	56	290									7	350	1190
										3	45	332										5	380	1570
									2	48	430											3.5	480	2050
								3	42	525												2.5	570	2620
							1	29	600													1.8	630	3250
						3	37	760														1.2	800	4050
					2	38	810															.9	850	4900
				4	46	950																.6	1000	5900
			4	46	1050																	.45	1100	7000
		4	46	1050																		.30	1100	8100
	4	46	1050																			.22	1100	9200
2	23	475																				.15	500	9700
16	284																					.11	300	10000
																						<.09	0	10000
20	310	530	1100	1100	1100	990	850	790	640	580	480	390	360	260	190	130	87	56	23	10	2			
10000	9980	9670	9150	8050	6950	5850	4860	4010	3220	2570	2000	1520	1130	760	500	310	180	91	35	12	2			

of-echo times time, on a radar display. So these tables compare the distribution of such area, with rainfall rate along the columns, with apparent rainfall rate along the rows.

Table II is for wavelength 5.7 cm, again for range 30 miles. It is clear from the diagonal form of the table that attenuation at wavelength 5.7 cm is much less than at wavelength 3.0 cm. Whereas at 3.0 cm, the 380 hours of rainfall rate 5 mm hr^{-1} were distributed among the whole range of apparent rainfall rates less than 5 mm hr^{-1}, at 5.7 cm no significant duration is distributed among apparent rainfall rates less than 2 mm hr^{-1}. Tables III and IV (appearing overleaf) complete the set by presenting distributions at the two wavelengths but this time for a 120-mile path to the target.

The scales for attenuation at wavelength 5.7 cm in Figs. 5 and 6 are based on attenuation at 5.7 cm being just one-sixth that at 3.0 cm. Tables II and IV thus depend in turn on this assumption. Its justification follows. In Fig. 7,

TABLE III - APPARENT MODIFICATION OF RAINFALL RATE DISTRIBUTION BY ATTENUATION AT 3.0-CM WAVELENGTH OVER 120-MI PATH

RAINFALL RATE (mm hr^{-1})																						RAINFALL		
<.09	.11	.15	.22	.30	.45	.60	.90	1.2	1.8	2.5	3.5	5	7	10	14	20	28	40	56	80	> 92	RATE	FREQ	CUM
											1	1	1	1	1	1	1	1	1	1		> 92	10	10
								1	1	1	1	1	1	2	2	2	1	1	1			80	15	25
			1	1	1	1	1	2	2	3	3	4	4	5	5	5	4	3				56	45	70
		1	1	1	1	2	2	3	4	4	6	6	7	8	9	8	6	1				40	70	140
		1	1	2	2	3	3	4	4	6	7	9	11	13	11	10	3					28	90	230
	1	1	2	2	3	4	6	6	8	10	14	17	20	20	20	6						20	140	370
3	1	2	3	4	5	6	7	9	12	18	22	32	32	34	10							14	200	570
5	2	3	4	5	7	8	10	13	22	27	43	51	54	16								10	270	840
4	3	4	5	8	9	12	14	24	32	53	74	80	28									7	350	1190
8	3	5	7	8	10	13	23	27	50	80	100	46										5	380	1570
15	5	7	9	11	15	24	31	50	82	140	91											3.5	480	2050
15	6	7	10	15	21	29	44	83	150	140												2.5	570	2620
17	7	9	14	23	35	45	90	170	220													1.8	630	3250
33	12	18	29	43	55	110	220	280														1.2	800	4050
44	19	31	46	60	120	230	300															.9	850	4900
80	36	54	70	140	270	350																.6	1000	5900
125	60	75	150	300	390																	.45	1100	7000
185	75	150	300	390																		.30	1100	8100
260	150	300	390																			.22	1100	9200
190	140	170																				.15	500	9700
190	110																					.11	300	10000
																						<.09	0	10000
1176	630	840	1040	1010	950	840	755	680	610	490	360	247	158	99	58	32	15	6	2	1				
10000	8820	8200	7360	6310	5300	4350	3510	2760	2080	1470	980	618	371	213	114	56	24	9	3	1				

TABLE IV - APPARENT MODIFICATION OF RAINFALL RATE DISTRIBUTION BY ATTENUATION AT 5.7-CM WAVELENGTH OVER 120-MI PATH

RAINFALL RATE (mm hr^{-1})																						RAINFALL		
<.09	.11	.15	.22	.30	.45	.60	.90	1.2	1.8	2.5	3.5	5	7	10	14	20	28	40	56	80	> 92	RATE	FREQ	CUM
																		1	3	5	1	> 92	10	10
																	1	4	8	2		80	15	25
																2	9	26	8			56	45	70
															2	12	37	19				40	70	140
														2	9	47	32					28	90	230
													2	16	66	56						20	140	370
												1	17	89	95							14	200	570
											1	19	100	150								10	270	840
										1	19	120	210									7	350	1190
									1	25	104	250										5	380	1570
								1	15	114	350											3.5	480	2050
							2	16	112	440												2.5	570	2620
						2	15	113	500													1.8	630	3250
					2	18	140	640														1.2	800	4050
				3	20	147	680															.9	850	4900
			3	24	173	800																.6	1000	5900
		4	26	190	880																	.45	1100	7000
	4	26	190	880																		.30	1100	8100
4	26	190	880																			.22	1100	9200
14	86	400																				.15	500	9700
60	240																					.11	300	10000
																						<.09	0	10000
80	360	620	1100	1100	1080	970	840	770	630	580	470	390	330	260	170	120	79	50	19	7	1			
10000	9920	9570	8950	7850	6750	5680	4710	3870	3100	2470	1890	1420	1030	700	450	270	160	77	27	8	1			

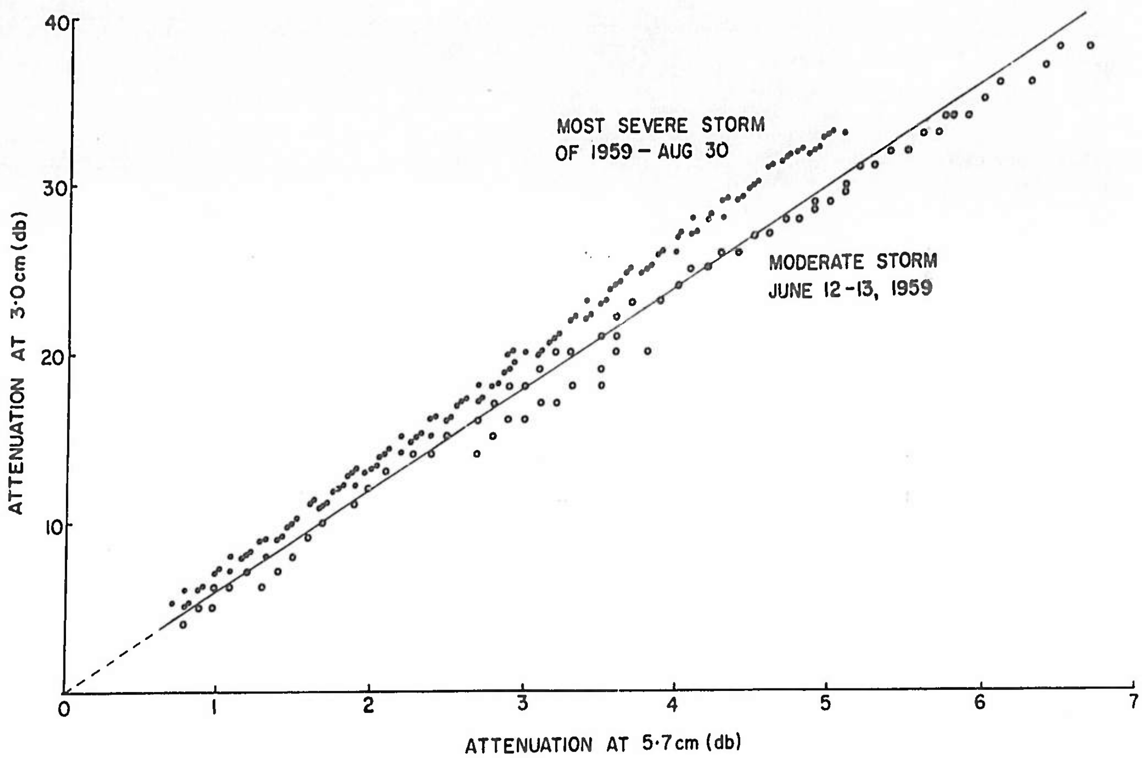


FIG. 7. Comparison of attenuation at wavelengths 3.0 cm and 5.7 cm.

attenuations at the two wavelengths are compared for two storms. Each point on the figure represents one "sample point", of which there was one to each 0.01 inch of rainfall. The attenuation at 3.0-cm wavelength from that point right out to the limit of the storm is plotted against attenuation along the same path at 5.7-cm wavelength. In the case of the moderate storm, chosen as a fairly typical case, the attenuation at 5.7 cm is about 1/6 that at 3.0 cm. In the most severe storm of the season the factor is about 1/6.7. This led us to adopt the factor 1/6 in converting all 3.0-cm calculations to wavelength 5.7 cm. The factor appears to be relatively independent of rainfall rate (R), in agreement with the proportionality of attenuation at 3.2 cm to $R^{1.31}$, and at 5.7 cm to $R^{1.17}$.

4. ATTENUATION AND HEAVY TARGET PRECIPITATION

Fig. 6 shows the close relation of attenuation to heavy target precipitation: for target rain of rate 40 mm hr^{-1}, at range 30 miles, half the cases have more than 10 db attenuation, 10% have more than 24 db (to quote the 3.0-cm figures). The further effect of increasing the range from 30 miles to 120 miles (Fig. 5) is to increase the attenuation (in db) by only about 20%. When the target rate of rainfall is reduced by a factor four, on the other hand, the 10 db attenuation drops greatly to 2 db, and the 24 to 12 db.

In Fig. 8, the redistribution of echo area (duration) due to 3.0-cm attenuation is shown for intervals of rainfall rate centred on 80, 20, 5 and 1.25 mm hr^{-1}. Light rainfall rates stand a good chance of being observed on the radar at their true value. Heavy rates are likely to be observed over a broad range of apparent rates, and stand very little chance of being observed at their true value.

The effect on heavy rain is rather worse with the "stepped grey-scale" of the CPS-9 radar at Montreal, because of the relatively wide intervals (factors four in rainfall): In each interval there is more rain close to the low-intensity

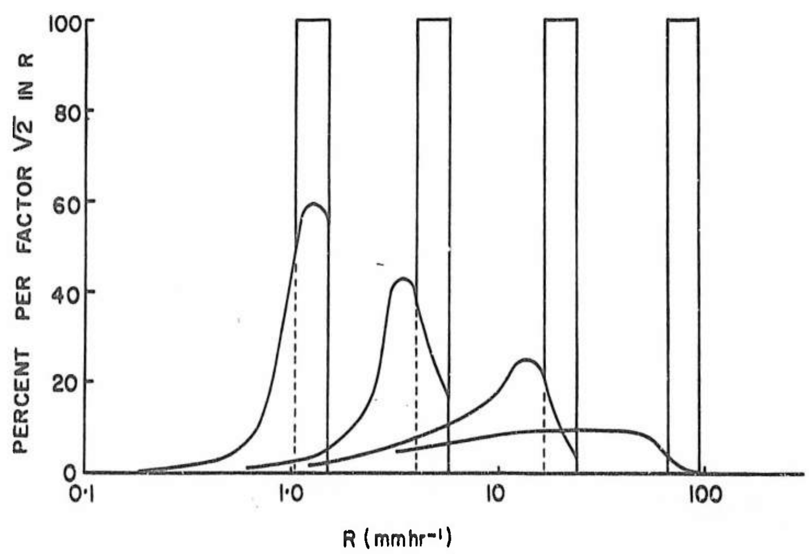


FIG. 8. Redistribution of echo duration by attenuation at wavelength 3.0 cm and range 30 miles.

boundary, where a little attenuation will shift it to the lower interval, than there is close to the high-intensity boundary, where more attenuation is required for an appreciable effect. The result is shown in Table V.

No Echo	Echo Shade 1	2	3	4	5	6	7	R(mm hr^{-1})
				(20)	(40)	(40)	(0)	>400 (7)
			20	40	40	0		100-400 (6)
1	0	8	25	50	16			25-100 (5)
1	1	7	48	43				6.4- 25 (4)
1	2	21	76					1.6-6.4 (3)
1	17	82						0.4-1.6 (2)
11	89							0.1-0.4 (1)

TABLE V

Echo shades arising from various rainfall rates (% of duration or area). Relevant to the McGill CPS-9 observing at 30 miles range. Performance at >400 mm hr^{-1} will be worse than shown.

The statistics used so far take into account losses in viewing all the heavy rain. But most of the attenuation to which signals from a storm are subjected occurs within that same storm. Thus the edge of the storm toward the radar may be observed with relatively little attenuation, so that there would be some warning of the storm's severity in the echo from the near edge, though a large part of the area of severity might not be observed as being severe. To produce some statistics on this aspect of observing severe storms we returned to the original storm intensity profiles, such as are shown in Fig. 3. Each storm with some record of a rate greater than 40 mm hr^{-1} was considered, a storm being treated individually if it was more than 30 miles from a neighbour. There were 21 such storms and, in order to augment the data, profiles were plotted for each storm viewed from both directions. Attenuated profiles were calculated for both 3.0- and 5.7-cm wavelengths, and all the profiles are shown in Appendix III.

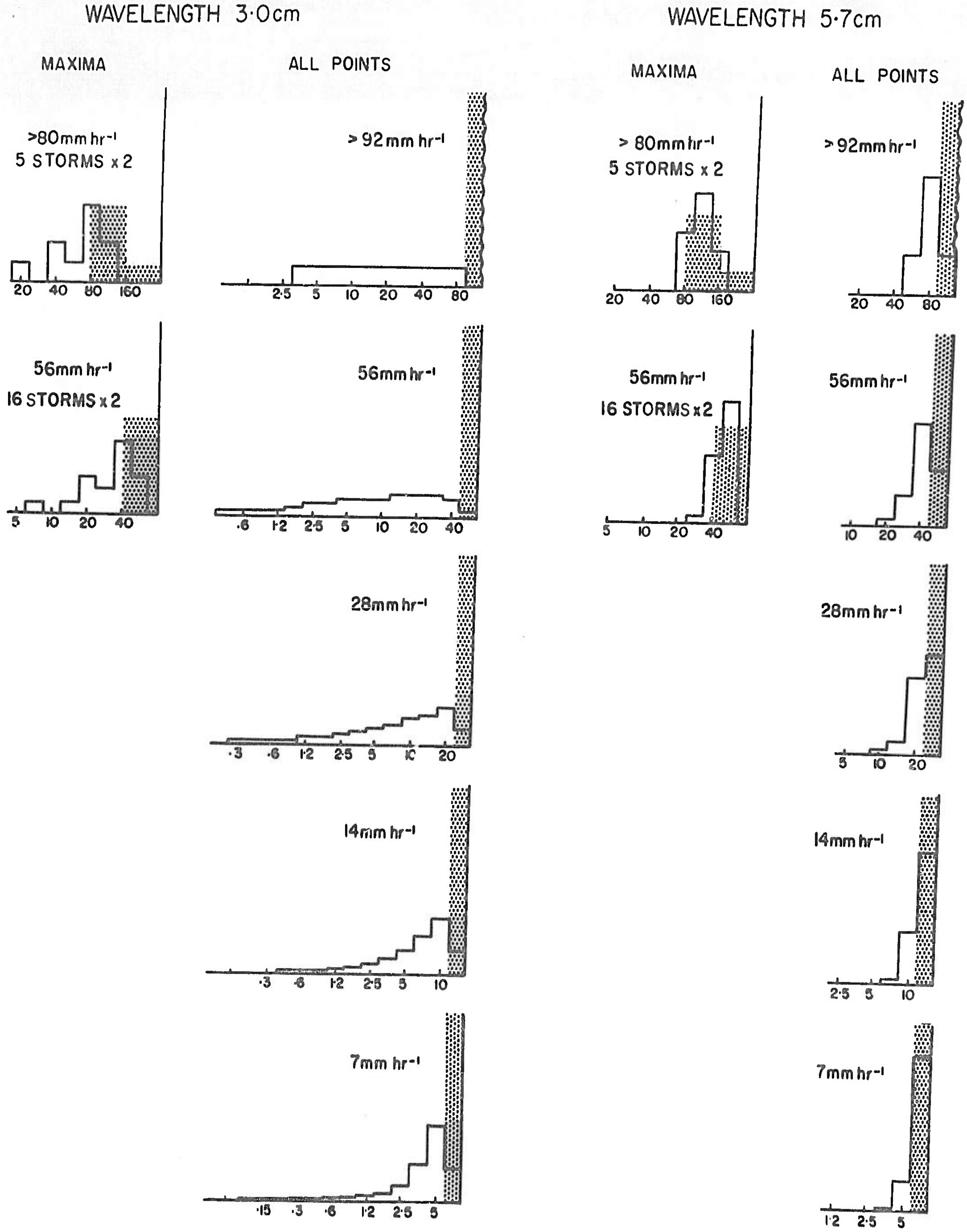


FIG. 9. Redistribution of storm maxima (apparent maxima versus true) compared with redistribution of all point data (much as in Fig. 8).

The maxima of the attenuated profiles (i.e. the maxima of apparent rainfall rate) are compared with the maxima of the unattenuated profiles in Fig. 9. Ten cases of true maxima greater than 80 mm hr^{-1} have been grouped together. In the first and third graphs at the top of the figure, the shaded portion indicates the distribution of the original maxima. The outlined histogram shows their redistribution, that is, the distribution of the maxima of apparent rainfall rate. The graph labelled "56 mm hr^{-1}" refers similarly to 32 cases with true maxima between 40 and 80 mm hr^{-1}. For comparison Fig. 9 also contains "all point" data from Tables I and II including extrapolation for data at rates >92 mm hr^{-1}. At wavelength 3.0 cm, the effect of attenuation on the maxima is seen to be less than that on "all point" data of the same rainfall rate. In fact, the redistributions of the maxima at 56 mm hr^{-1} and >80 mm hr^{-1} resemble the "all point" distributions at 7 and 14 mm hr^{-1}: That is, the effect of attenuation on the maxima is much like its effect on "all point" data of one-eighth the rainfall rate. At wavelength 5.7 cm, there is less difference, only a factor two or four between the intensity of maxima and the intensity of "all point" data having the same redistribution.

To judge the effect of stepped grey-scale interpretation, at 3.0 cm, these redistributions of maxima were arranged after the fashion of Table I. By grouping the quantities in this arrangement, Table VI was produced which is analogous to Table V for "all point" data. All the effects at 5.7 cm are relatively

Max. Echo Shade 3	4	5	6	7	Max. R(mm hr^{-1})
		(4)	(78)	(18)	>400 (7)
	4	78	18		100-400 (6)
4	50	46			25-100 (5)

TABLE VI

The greatest echo shade associated with each maximum rainfall rate (% of cases or area). McGill CPS-9 observing at 30 miles range. Performance at >400 mm hr^{-1} will be worse than shown.

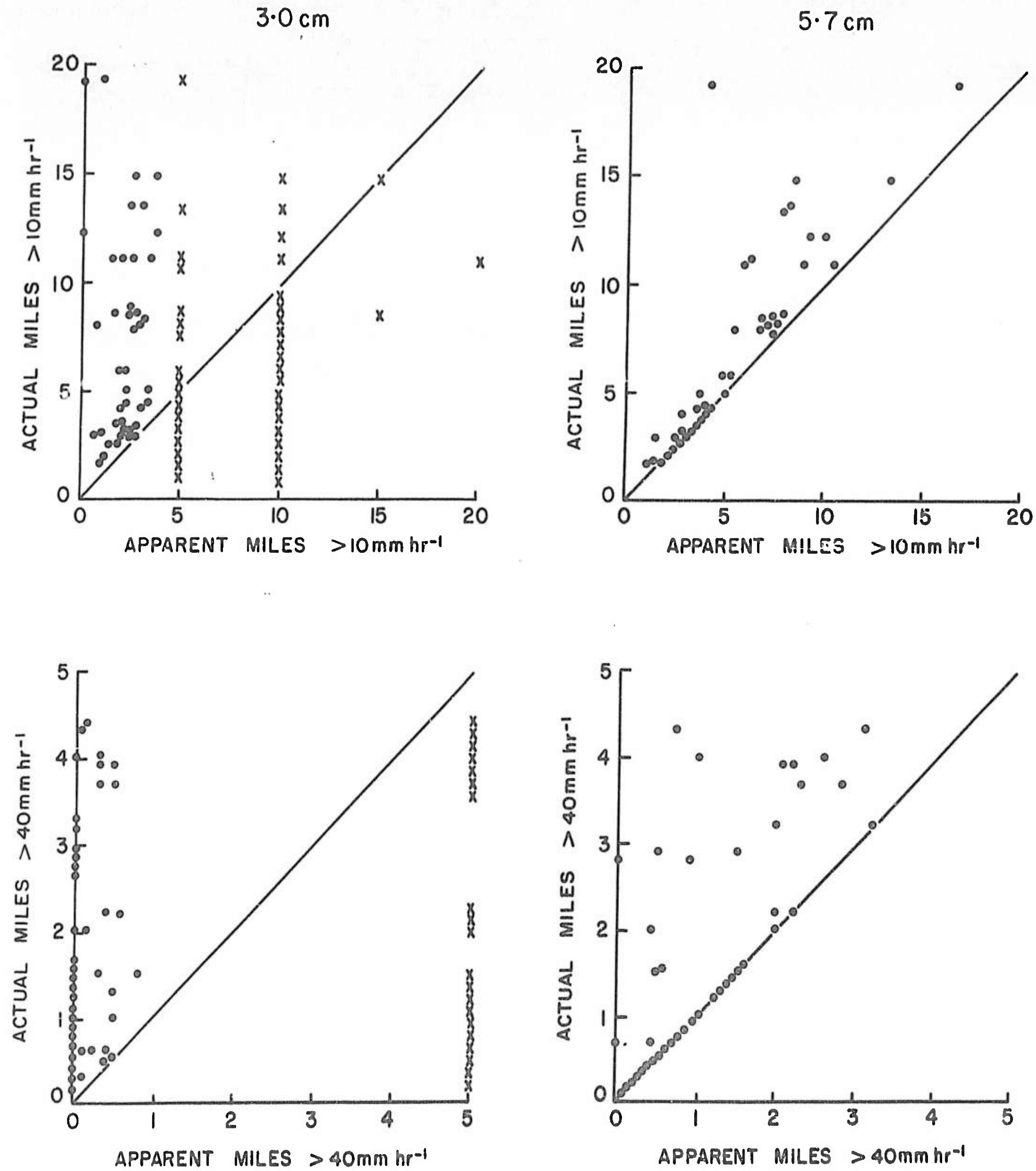


FIG. 10. The full circles compare actual and apparent storm lengths, for two wavelengths and two threshold rainfall rates. For wavelength 3.0 cm, the crosses are for results using a five-mile grid. There should be additional crosses superimposed on all full circles lying on the ordinate axis.

small, and a negligible number of data are reproduced at less than half their true intensity.

In Fig. 10, consider first the dots. They plot the length of that part of a storm exceeding a fixed rainfall rate against the length apparently exceeding that rate. At the 10 mm hr^{-1} level (wavelength 3.0 cm), four miles is the greatest apparent length, although actual lengths extend up to nearly 20 miles. At 40 mm hr^{-1}, the actual lengths extend to 4.5 miles, the apparent lengths to only 0.8 miles; half the actual cases don't appear at all, and so are plotted on the ordinate axis. At neither rate is there much correlation between the actual and apparent lengths. Even at wavelength 5.7 cm, for rates greater than 40 mm hr^{-1}, half the cases show no correlation.

The crosses in Fig. 10 give the results when a five-mile grid is used: If the fixed rate is exceeded anywhere within a given five-mile unit, then the whole unit is counted as length above that rate. At wavelength 3.0 cm the average lengths are nicely restored by use of the grid:

	10 mm hr^{-1}	40 mm hr^{-1}
Actual length	2.3 mi	1.9 mi
Apparent	2.3 mi	0.17 mi
Apparent with grid	8.0 mi	2.5 mi

At 10 mm hr^{-1}, this is a useful method of compensation. At 40 mm hr^{-1}, however, half the apparent lengths are zero, and so cannot be restored by this compensation. At wavelength 5.7 cm attenuation is so much less that use of the grid considerably exaggerates the areas of high intensity.

Our final approach to intense individual storms was to correlate the maximum attenuation that would be effected by the storm as it passed by with the maximum rate of rainfall in the storm. This was done first for the storms

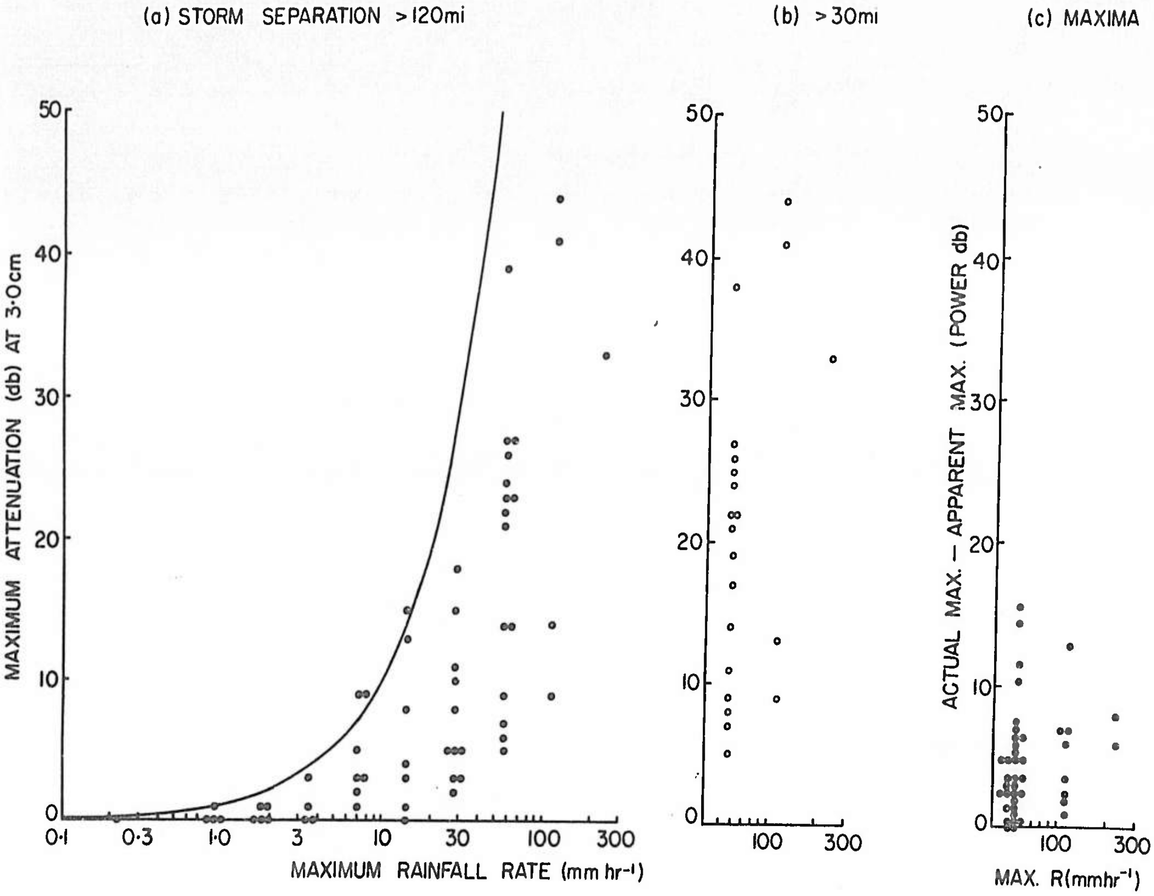


FIG. 11. The maximum attenuations of parts (a) and (b) are much greater than the amount by which the apparent maxima fall below the actual.

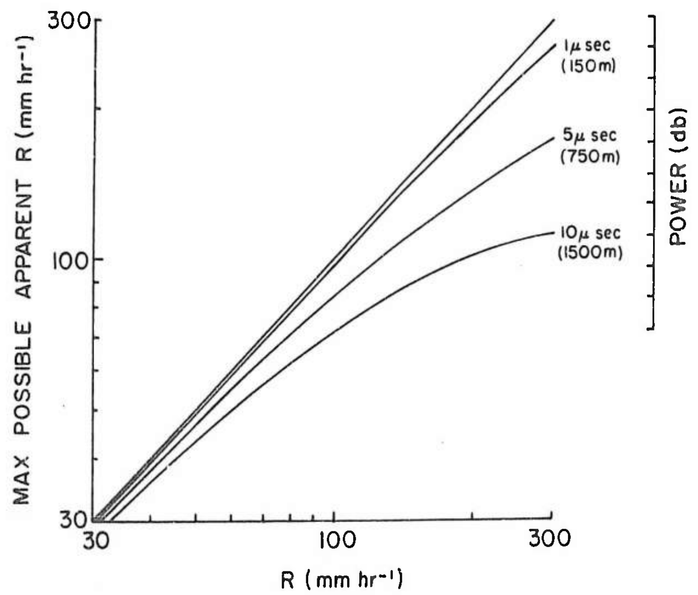


FIG. 12. Effect of pulse length at wavelength 3.0 cm.

with maximum rates of rainfall greater than 40 mm hr^{-1}. As above, a storm was treated individually if it was more than 30 miles from a neighbour. Displayed in Fig. 11b, the correlation shows four storms out of 21 with more than 30 db attenuation at 3.0 cm, 16 with more than 10 db. In order to extend this approach to all of the summer's rain, it was found desirable to require 120 miles separation of storms rather than the 30 miles so far used. Plotted in Fig. 11a, on this basis, the correlation for high intensities shows little difference from Fig. 11b. A few of the less intense storms give attenuation greater than 10 db, and the total exceeding 10 db is now 20 of the 58 storms. An interesting feature of Fig. 11 is the fact that in very few storms does the attenuation in db exceed the greatest rainfall rate in mm hr^{-1}. This is shown by the curve plotted in the figure.

We have already noted that the difference between apparent and actual storm maxima is not as severe a reduction as that effected by attenuation on the areas of intense cores. To end the section on a less dire note than sounded in Fig. 11, (a) and (b), we have added in (c) the data of Fig. 9, rearranged for comparison.

5. EFFECT OF PULSE LENGTH

More than fourteen years ago Ryde (1946) was disturbed at the drastic attenuation within a pulse length at heavy rainfall rates. The whole analysis in the present study assumes an infinitesimal pulse length. Actually, with a finite pulse length the received power is an average of the signals from a range interval of half a pulse length. So the maximum power received from a storm of constant rainfall rate is that scattered from the half pulse length of the storm nearest the radar. Fig. 12 is a plot of the maximum apparent rainfall rate that can be observed with wavelength 3.0 cm in a storm of given actual rainfall

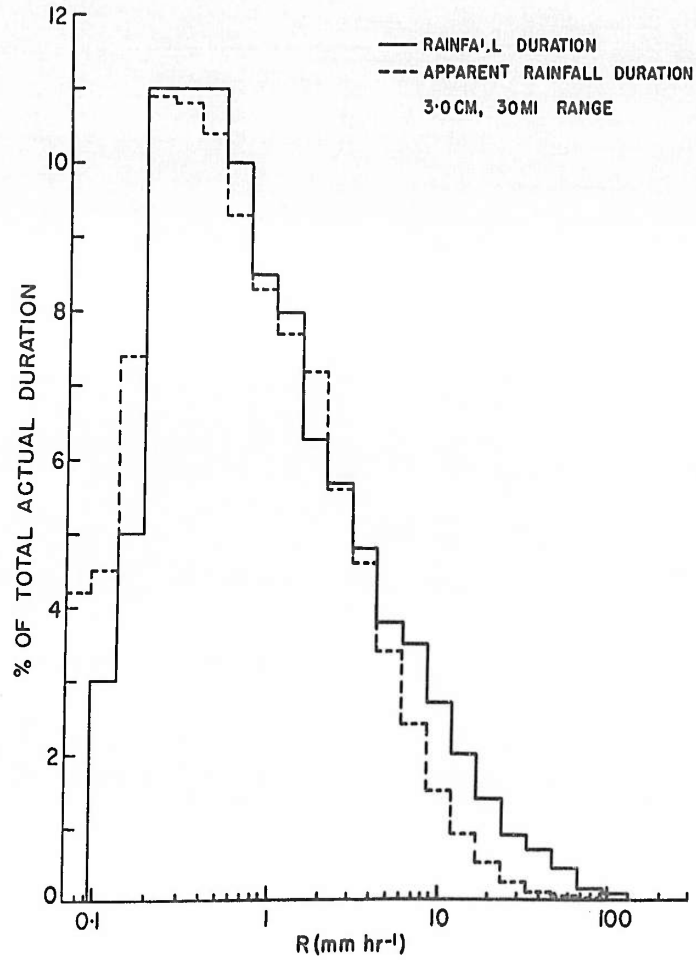


FIG. 13. Rainfall duration as a function of rainfall rate.

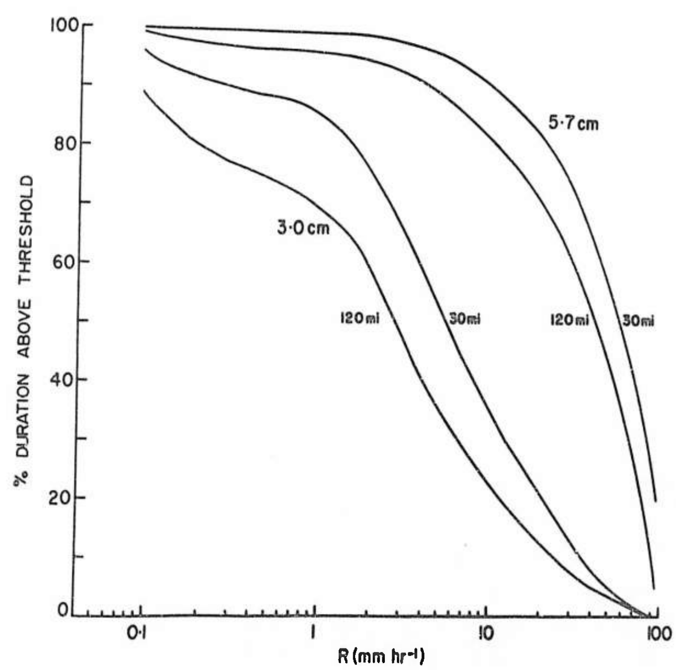


FIG. 14. The ratio of apparent duration exceeding a given rainfall rate to actual duration exceeding that rainfall rate.

rate. Perhaps the most serious effect of this pulse length averaging on the present work is to produce optimistic statistics on the distribution of apparent maximum rainfall rates (previous section). Referring to Fig. 3 for a moment, the one mile of maximum rate 112 mm hr^{-1} seems to be reduced to an apparent rate 5 mm hr^{-1}: In fact, the slender peak is smoothed out and, with 5 μsec pulse, would be observed at 4 mm hr^{-1}. The effect is also present in the rest of the work, but it is probably true to say that for pulse lengths up to about 5 μsec and for rainfall rates up to about 100 mm hr^{-1} the effect is not serious.

6. DETECTION AND MEASUREMENT OF RAINFALL

On a plot of echo area or rainfall duration against rate (Fig. 13) the shift due to attenuation is at first glance almost negligible. The solid line (corresponding to the duration curve of Fig. 2 and to the frequency column of Tables I to IV) is the distribution of echo-area with intensity if there were no attenuation. The broken line (second-from-bottom row of Table I) is the distribution as modified by attenuation at 3.0 cm. Or it can be said that the solid curve is rainfall duration, and the broken one apparent rainfall duration. The discrepancy between the high-intensity tails assumes great significance because one attributes a special importance to heavy and very heavy precipitation, and to being able to distinguish it as such.

In Fig. 14, the abscissa is threshold sensitivity of a radar, with specified wavelength and range. The ordinate is the fraction of rainfall duration above that threshold that is detected as echo area. The CPS-9, with a threshold of the order of 0.1 mm hr^{-1} (at long range) misses very little. With the same 3.0-cm wavelength and lower power, for a threshold of 10 mm hr^{-1}, only from 20% to 40% (depending on the range through which attenuation is suffered) of the rain occurring at rates above 10 mm hr^{-1} is detected. Surprisingly to

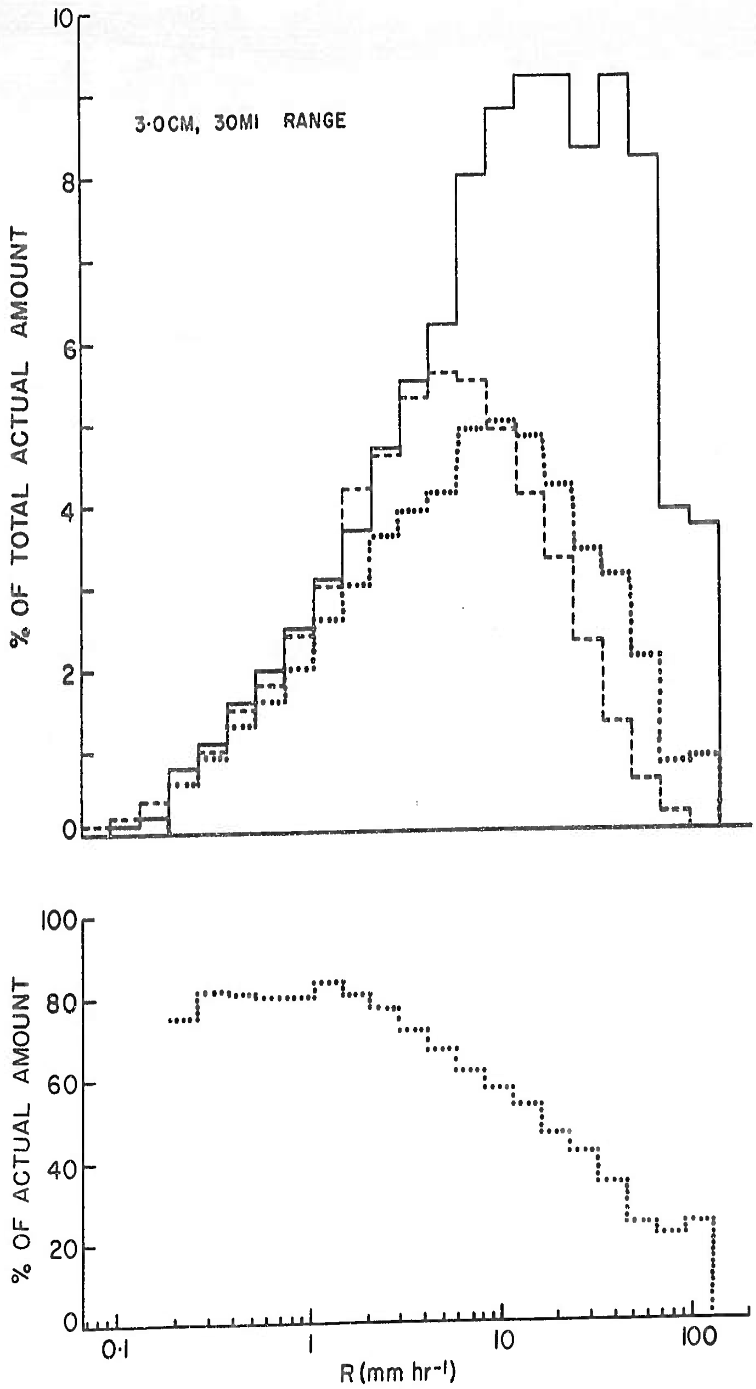


FIG. 15. Above: Rainfall amount as a function of rainfall rate.
Below: Ratio of apparent rainfall amount (dotted curve above) to actual rainfall amount (solid curve above) as a function of rainfall rate.

us, performance at 5.7 cm starts falling off seriously at 5 mm hr^{-1}, and is in the 20% to 40% range mentioned above for threshold sensitivity 80 mm hr^{-1}.

Fig. 14 is relevant to radars using the "hole-punching" technique to provide a single high-intensity contour in addition to the limiting-sensitivity outline. In this technique, the user can set-in a relatively high threshold, and everything above this threshold is reproduced black instead of white, thus giving black cores to intense storms. As the threshold is increased across the rainfall-rate scale of Fig. 14, the <u>area</u> of the "hole" or dark core falls off, of course. The relevance of Fig. 14 is that the area becomes a progressively smaller fraction of what it should be, reaching one-third at about 10 mm hr^{-1} for 3.0 cm and at 80 mm hr^{-1} for 5.7 cm. That the average reduction at 10 mm hr^{-1} for 3.0 cm is a factor three is suggested in Fig. 10, where it can also be seen that the reduction ranges up to a maximum of a factor 30.

So far the discussion has been confined to assessing the effects of attenuation on duration (or areal extent) of rainfall. It is possible to assess the effects on measurement of rainfall amount by turning to Tables I to IV. The solid curve of Fig. 15 shows the distribution of rainfall amount with rate and is similar to the smoothed "amount" curve of Fig. 2. In fact it is the product of the rainfall duration and the corresponding rate as found in the right-hand margin of Tables I to IV. The broken curve shows the distribution of apparent amount that would be observed at a point at 30 miles range, with wavelength 3.0 cm. Its ordinate values are taken from Table I, being the products of rates (top row) and durations (second-from-bottom row). Alternatively, the ordinate function is the product of the ordinate function in Fig. 13 with R. The dotted curve is something else again. It is derived from Table I as the sum of duration-rate products in any row: That is, for a given

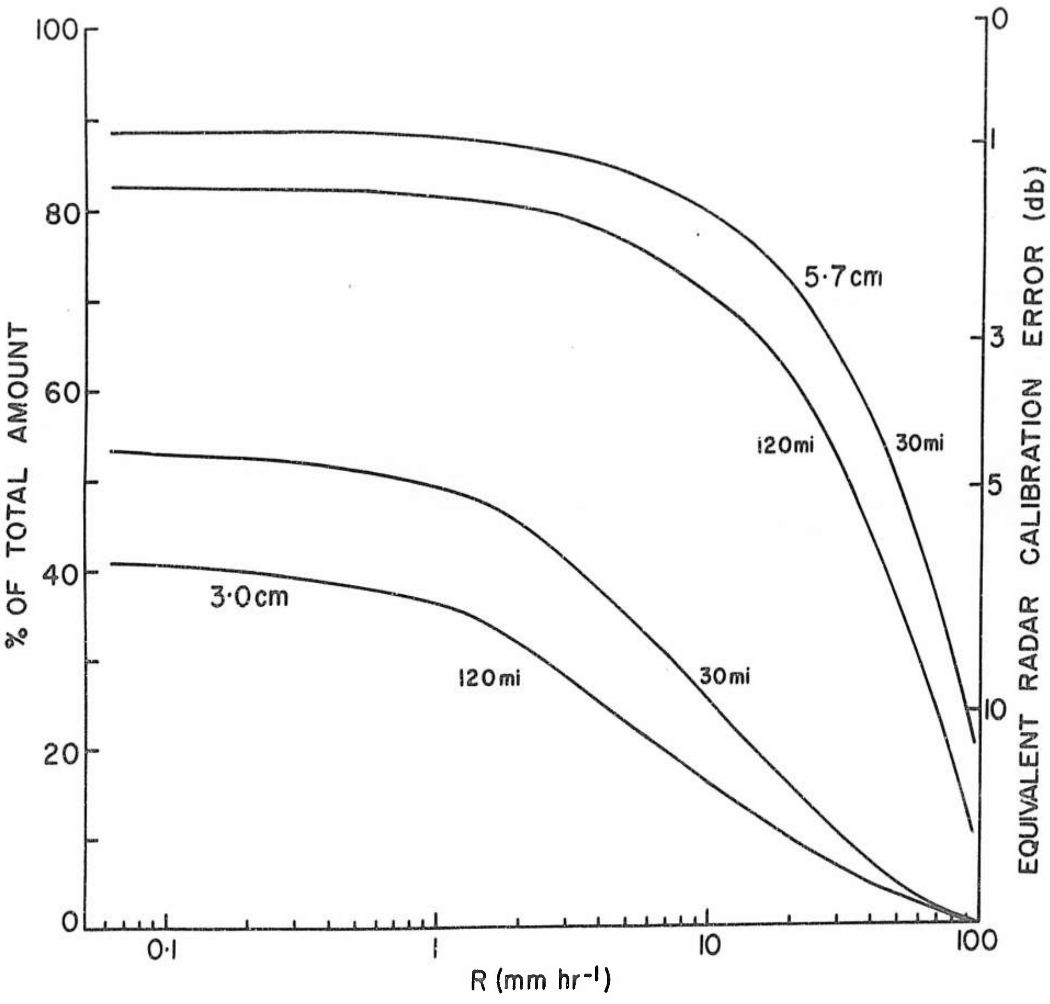


FIG. 16. The ratio of apparent amount exceeding a given rainfall rate to actual amount exceeding that rainfall rate.

rainfall rate, it indicates the total amount of rain measured by the radar, due to rain that actually has that rate. The radar interprets this rainfall as being distributed over a wide, and generally lower, range of rates. Finally, the lower graph shows the ratio of this measured rainfall to the actual amount of rainfall as a function of the actual rate of rainfall. We were surprised to find how high these efficiencies are.

Proceeding further, Fig. 16 shows for any threshold rate of rainfall the fraction of rainfall, due to rain falling at rates above this threshold, that will be measured by radars of specified wavelength at specified ranges. The right-hand scale of ordinates gives the equivalent errors in radar calibration, i.e. the errors, overestimating performance, that would effect the same error in rainfall measurement. For a season's rain and for a threshold sensitivity something like 1 mm hr^{-1}, the wavelength 5.7 cm would seem to suffer no more from attenuation (on this basis) than from limitations of calibration. Even with a 3.0-cm radar, it is quite conceivable to have calibration errors comparable with those indicated for this threshold, in Fig. 16. But perhaps this is not a fair comparison since a radar calibration error which is consistent would soon be detected and remedied. Alternatively, against this calibration-error scale, the ordinate values represent an average attenuation over specified paths to all target rainfall falling at rates above thresholds on the abscissa. So at wavelength 3.0 cm we estimate that average attenuation for thresholds of 1 mm hr^{-1} or less over a 30-mile path is about 5 db and over 120 miles is 7 db.

Finally, proceeding shower by shower, how does the amount of rain in a shower, or that would be collected from a shower, compare with the amount indicated by radar? That is to say, how much lower is the latter, because of

attenuation? In Fig. 17 (for showers with maximum rainfall rate greater than 40 mm hr^{-1}), at wavelength 3.0 cm the actual rainfalls practically all lie within a factor six of the observed or apparent rainfalls. At wavelength 5.7 cm, the factor is 1.7. The actual amounts range from 3 to 40 mm per shower. The actual total amount for the season's heavy showers (21 showers x 2) was 473 mm; apparent amounts at 3.0 cm totalled 144 mm, and at 5.7 cm totalled 349 mm.

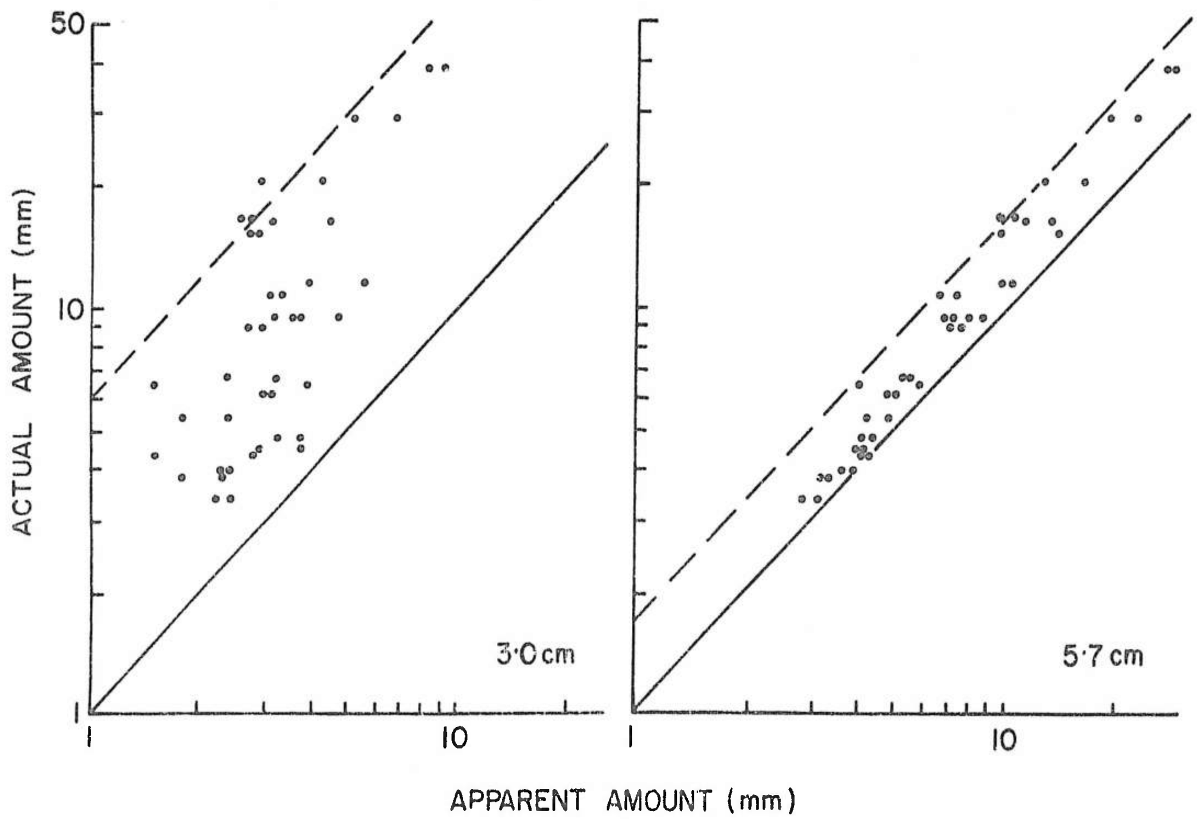


FIG. 17. Comparison of actual rainfall amount in each intense shower with the apparent amount.

7. SUMMARY AND CONCLUSIONS

Data from one raingauge for one five-month summer period have been combined with velocity information from radar records, to provide synthetic storm structures. Most of these storms were showers or thunderstorms, with relatively little continuous rain. Attenuation data have been calculated at wavelengths 3.0 cm and 5.7 cm. (Regarding the former, it is recognized now that 3.2 cm is a more relevant wavelength; attenuation is 20% less at 3.2 cm than at 3.0 cm.) In this summary, results are given both for wavelength 3.0 cm and (following in brackets) for wavelength 5.7 cm.

Statistically, the amount of attenuation has been found to vary greatly with the intensity of the target rain. It is this correlation of attenuation with rate of target rainfall that makes attenuation at 3 cm troublesome and insidious.

For cases of target rain of rate 40 mm hr^{-1} at range 30 mi, half the cases have more than 10 db (1.5 db) attenuation, 10% of them have more than 24 db (4 db). A decrease in intensity of the target rain by a factor four makes the statistics much better, whereas an increase in the range of a factor four makes them little worse.

If 0.1 mm hr^{-1} is taken as a threshold, then for range 30 mi the map-area indicated as having rates greater than this is 96% (99.8%) of what it should be, and the amount of rainfall at greater rates is 53% (88%) of what it should be. But if 40 mm hr^{-1} is taken as a threshold, then the map-area indicated as having greater rates is only 8% (63%) of what it should be, and the amount indicated is only 7% (56%) of what it should be.

Most attenuation is due to heavy rain in the same shower as the target rain. The raingauge whose data were considered recorded rates greater than 40 mm hr^{-1} from 21 showers during the five-month period. The total attenuation

effected by a single shower, looking all the way through it, is most impressive: Of the 21 showers, half gave attenuations greater than 20 db (3 db), only a quarter gave less than 10 db (1.5 db). It could be argued that the area of the shadows cast by these storms was too small for this finding to be significant, since only one of these shadows contained a storm comparable with those that cast the shadows. Even so, at 3 cm the shadows are embarrassingly deep.

The effect of attenuation within a shower on the reproduction of the shower itself is to reduce both the maximum value of intensity and the area of the high-intensity core, while tending to displace the maximum and core slightly toward the radar, and generally distorting the pattern of the storm (in line with Atlas and Banks, 1951). The reduction in intensity from the true maximum to the differently-located apparent maximum is considerably less than the attenuation at the position of the true maximum. In the 42 cases obtained by treating each of the 21 showers mentioned above from two directions, half the maxima were down by the equivalent of 4 db (1 db), and 10% were down by the equivalent of 11 db (3 db). This is the happiest finding concerning attenuation at wavelength 3 cm: When isolated showers are judged by the maximum recorded intensity within the shower, and not by the size of core, the effect of attenuation is usually not very bad. As for the cores, statistics already referred to indicate an average reduction in area at threshold 40 mm hr^{-1}, to 8% (63%) of what it should be.

The 21 showers which reached maximum rates greater than 40 mm hr^{-1} delivered between 3 and 40 mm of rain to the gauge. Radar records would never have indicated more than 9 mm (28 mm) and the true amount was anything up to six times (1.7 times) what the radar would have indicated, because of attenuation.

The possibility of using an attenuating wavelength and correcting for attenuation exists, although the behaviour of a correcting system is bound to

be rather subtle. If it proves practicable, the present report provides evidence that correction should only be applied to 3-cm radars when they are sensitive down to something like 0.1 mm $hr.^{-1}$; otherwise, there will be intense and significant precipitation whose signals will be lost before they can be corrected.

If the radar displays target strengths as one of a small number of discrete values, the effect of attenuation on target strength is made worse. Thus for the case of 40 mm hr^{-1} target rain at range 30 miles, half the cases suffered attenuation 10 db (1.5 db), equivalent to a factor four in rainfall rate. But of all the cases in the factor-four interval 25 to 100 mm hr^{-1}, the fraction shifted down out of that interval was 84% (24%), and 34% (0%) were shifted by more than one factor-four interval. A similar effect is noted when apparent maximum rainfall rates for showers are compared with true rates. Another conventionalization of the radar display is to denote for the whole of a unit area, say five miles by five miles square, the maximum rainfall rate within that square. This tends to compensate for the reduced areas of cores.

Obviously, attenuation is much less troublesome at 5.7 cm than at 3 cm. But measured values of rainfall rate and amount of rainfall can still be out by considerable factors. The present evidence supports an old recommendation, that to avoid distortion one should go from 3 cm to 5.7 cm, while for truly quantitative operation the further step to 10 cm can be justified.

It should be noted that we have only considered summer showers at Montreal. Extensive moderate rain such as occurs in the fall and spring is a different matter, and has not been considered.

8. TECHNICAL NOTE OF THE WORLD METEOROLOGICAL ORGANIZATION

"Use of Ground-Based Radar in Meteorology" is the title of a W.M.O. Technical Note (Jones et al, 1959). "It appears" to the authors of that report "that a wavelength of 3.2 cm may be seriously affected occasionaly in tropical areas, but only rarely outside these areas". Earlier, they say "that in the British Isles..... an attenuation of 10 db may occur once or twice a year at a wavelength of 3.2 cm." References are not given, but apparently Jones et al have drawn on climatological data presented and considered by Ryde (1946) (See Appendix II). In the light of our present study, and of our experience leading to that study, and indeed of publications overlooked by Jones et al (as, for example, Hitschfeld and Marshall, 1954) the suggestion that wavelength 3.2 cm is rarely affected seriously by attenuation, outside the tropics, is dangerously misleading.

REFERENCES

Atlas, D., and H.C. Banks, 1951: The interpretation of microwave reflections from rainfall. J. Meteor. 8, 271-282.

Donaldson, Ralph J., 1958: Analysis of severe convective storms observed by radar. J. Meteor. 15, 44-50.

Gunn, K.L.S., and T.W.R. East, 1954: The microwave properties of precipitation particles. Quart. J.R. Meteor. Soc., 80, 522-545.

Haddock, F.T., 1948: Scattering and attenuation of microwave radiation through rain. Naval Res. Lab., Washington (unpublished manuscript).

Hitschfeld, W. and J. Bordan, 1954: Errors inherent in the radar measurement of rainfall at attenuating wavelengths. J. Meteor. 11, 58-67.

Hitschfeld, W. and J.S. Marshall, 1954: Effect of attenuation on the choice of wavelength for weather detection by radar. Proc. I.R.E., 42, 1165-1168.

Jones, R.F., et al, 1959: Use of ground-based radar in meteorology. W.M.O. Tech. Note No. 27, 80 pp.

Legg, T.H., 1960: The quantitative display of radar weather patterns on a scale of grey. Sci. Rep. MW-31, Contract AF-19(604)-6617, Montreal, McGill Univ., 113 pp.

Marshall, J.S., and K.L.S. Gunn, 1960: Wide dynamic range for weather radar. Beiträge zur Physik der Atmosphäre (in press).

Ryde, J.W., 1946: The attenuation and radar echoes produced at centimetre wavelengths by various meteorological phenomena. Meteorological Factors in Radio-Wave Propagation, (London, Physical Society, 1946), 169-188.

Ryde, J.W. and D. Ryde, 1945: Attenuation of centimetre and millimetre waves by rain, hail, fogs and clouds. G.E.C. Res. Lab., London, Rep. No. 8670.

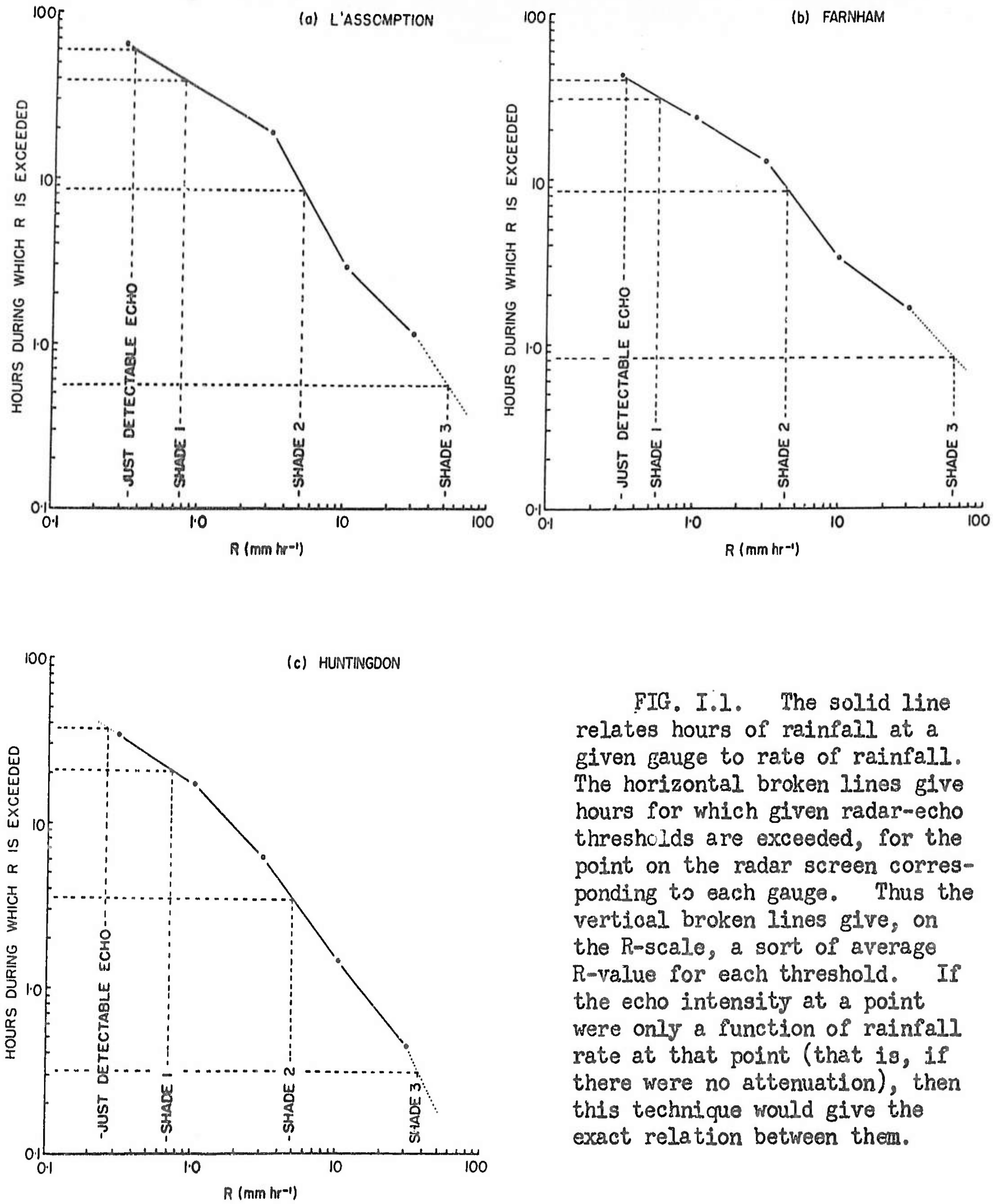


FIG. I.1. The solid line relates hours of rainfall at a given gauge to rate of rainfall. The horizontal broken lines give hours for which given radar-echo thresholds are exceeded, for the point on the radar screen corresponding to each gauge. Thus the vertical broken lines give, on the R-scale, a sort of average R-value for each threshold. If the echo intensity at a point were only a function of rainfall rate at that point (that is, if there were no attenuation), then this technique would give the exact relation between them.

APPENDIX I

ATTENUATION ESTIMATES FROM RADAR RECORDS

In this appendix is a brief account of an earlier study of attenuation from radar records. It was the limited success of this study that led to the present work.

Radar echo intensity at 7500 ft over and within a mile of three stations was compared with the rainfall rates at those stations during summer storms, June 1 to August 22, 1959. The stations were, with bearings from the radar:

L' Assomption	(30°, 29 mi)
Farnham	(107°, 39 mi)
Huntingdon	(220°, 33 mi)

Echo intensities on the CPS-9 3.2-cm radar were displayed as shades on a stepped grey-scale. The steps between successive shades were set to be 9.6 db apart in received power, or a factor four in rainfall rate. Rainfall rates were measured by tipping-bucket gauges which recorded the accumulation of each 0.01 inch of rainfall.

Because the echo intensity is a function both of the rainfall rate at the target and (through attenuation) of the rate along the path to the target, there is little to be gained by detailed minute-by-minute comparison of echo intensity with target rainfall rate. Instead, Fig. I.1 was prepared for the season at the three stations. The ordinates are the total durations for which rainfall rates given on the abscissae were exceeded. The heights of the vertical lines show the duration for which a given echo intensity was exceeded, and hence imply the average rainfall rate corresponding to the given intensity. It can be seen that the factors between the grey shades are considerably greater

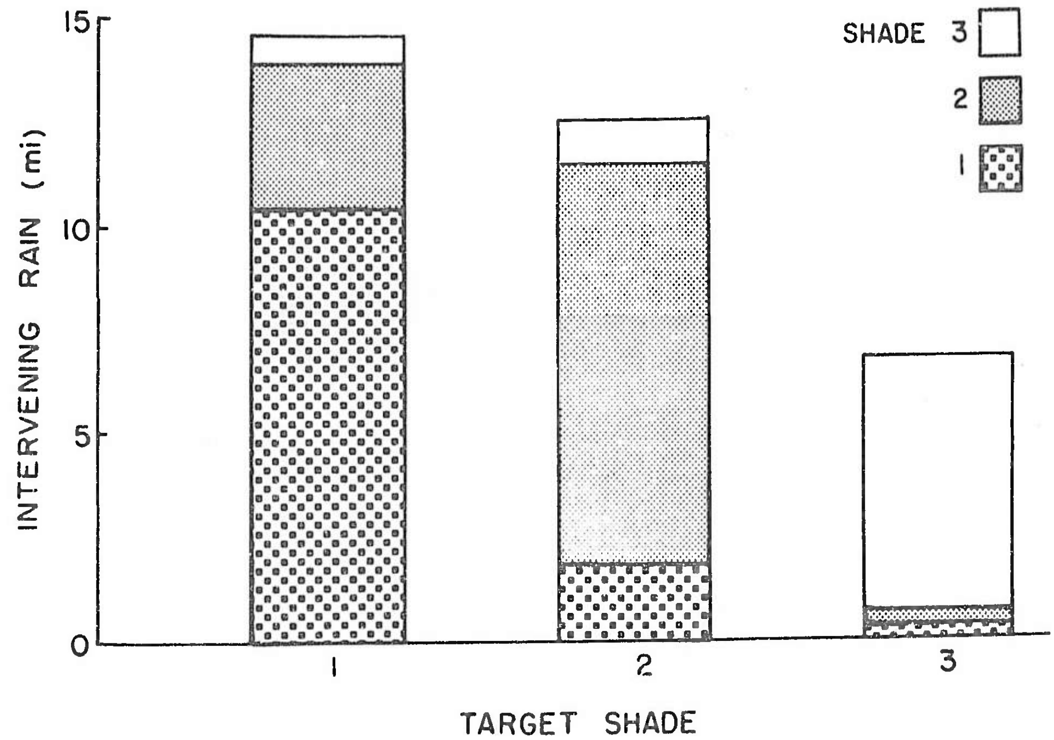


FIG. I.2. The average lengths of each echo shade along a 33-mile radar path to a target as a function of the echo shade at the target.

than the factors four in rainfall rate to be expected in the absence of attenuation. In fact, the factor averages about eight for the shades 1, 2 and 3 at the three stations.

Two points are worth making. First, the expected attenuation over the 33-mile path must increase as the target rainfall rate increases, in order to account for the fact that the grey shades are separated by a factor greater than four in rainfall rate. Second, there seems to be little effect of path orientation, for the statistics of the three stations are quite similar. Both these points have been exploited in the present work.

Further support for the notion of attenuation increasing with target rainfall rate is shown in Fig. I.2, which is drawn from radar records of the period 9 August to 22 August, 1959. It shows, for each echo shade at a station, the expected length of each echo shade intervening between the radar and the station. The greatest length is, in each case, seen to be of the same shade as that at the target.

It was hoped to gather attenuation statistics from this sort of work with radar records. It soon became apparent that this was impracticable, largely because of the extreme dependence of any such statistics upon small amounts of high-intensity echo. For instance, at wavelength 3.0 cm, one mile of echo shade 3 effects as much attenuation as nearly 50 miles of shade 1. Hence it seemed much more satisfactory to acquire statistics by the method used in the present work.

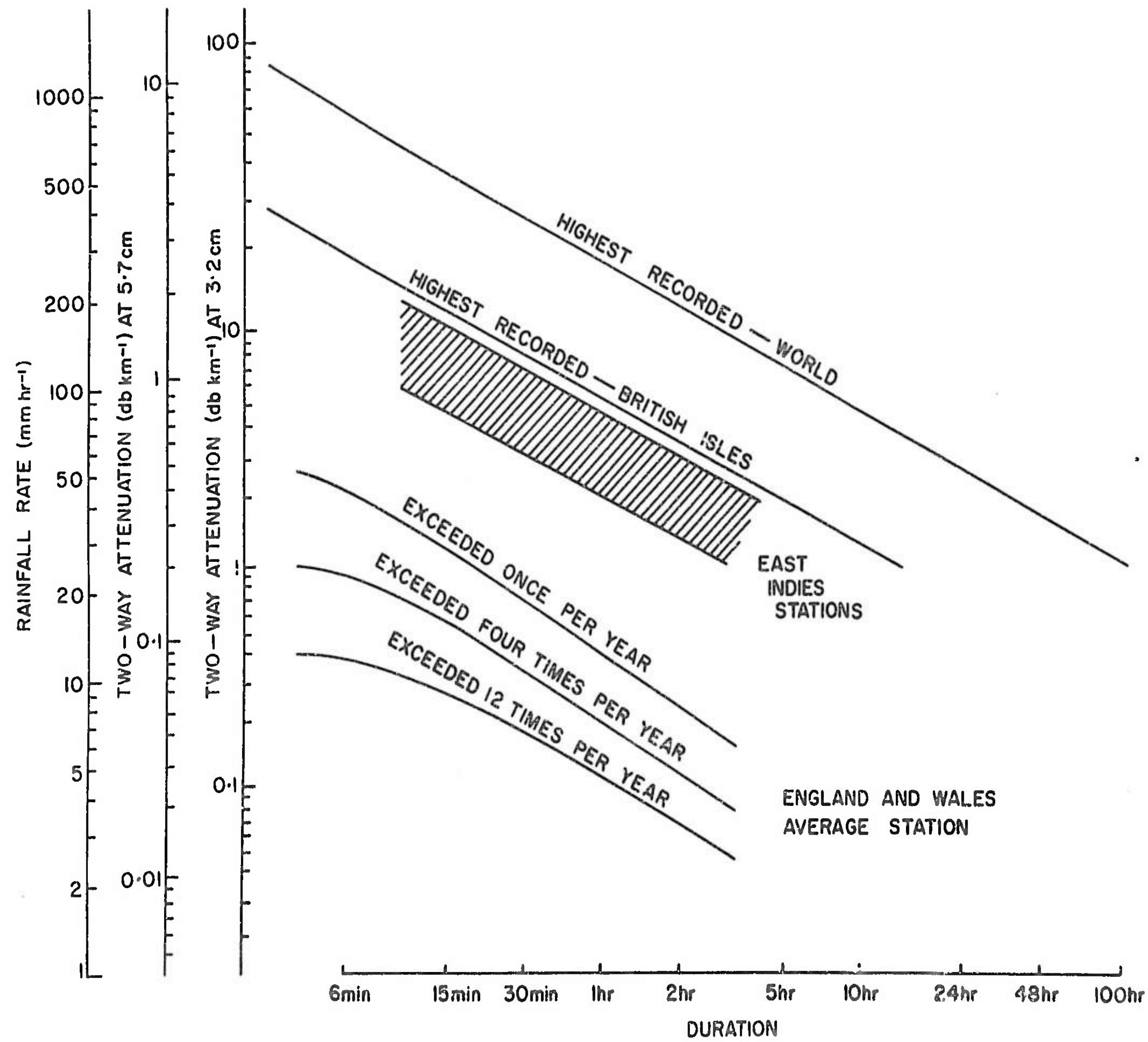


FIG. II.1. Rainfall intensity for various durations (from Ryde's (1946) Fig. 8). We have added scales of two-way attenuation from the data of Fig. 1a, much as they were added by Ryde and Ryde (1945).

APPENDIX II

THE METHOD OF W.M.O. TECHNICAL NOTE No. 27

Ryde (1946) collected statistics showing the expected frequency of given precipitation rates at various geographical locations, which are presented in Fig. II.1 (his Fig. 8). A rather more comprehensive discussion of these statistics appears in an earlier report (Ryde and Ryde, 1945) in which attenuation at several wavelengths is also related to precipitation rates. In Fig. II.1 we have added attenuation (two-way, in db km^{-1}) from the data shown in our Fig. 1a, much as they were added in that report.

Jones et al (1959), use this sort of precipitation statistics to estimate the only other attenuation statistics that have come to our notice. Unfortunately, it is difficult to interpret attenuation statistics, derived in this manner, in a meaningful or useful manner. Any such interpretation leads to a statement of what may be called absolute frequency; that is of the form "an attenuation of x db occurs n times a year". A statement of absolute frequency must be made with reference to two conditions in order to be meaningful: These require the specification of a time interval and the specification of an azimuth interval which separate independent occurrences. Even if these conditions are met the statement is not especially useful.

Throughout the present work we have tended to use relative frequencies of the form "an attenuation of x db along the radar path to a point occurs a fraction N/n of the time that it is raining R $mm\,hr^{-1}$ at the point".

Nevertheless it is readily possible to derive a limited form of absolute frequency from this present work. Each locus of Figs. 5 and 6 refers to a specific rainfall rate, so the ordinate can equally well be read as fractional duration or as fractional amount of rain. Hence by summing the fractional durations over all rainfall rates in the proportions in which they are observed

to occur in nature, the total duration of any given attenuation can be computed. In Fig. II.2 these durations are presented for wavelengths 3.0 cm and 5.7 cm in terms of 100 hours of rainfall at rates exceeding 0.3 mm hr^{-1} for targets viewed at various ranges. At each of the three gauges in the Montreal area referred to earlier, the number of hours of rainfall greater than 0.03 mm hr^{-1} was rather greater than 100 hours. Hence we may conclude, for example, that in the Montreal area, at wavelength 3.0 cm, an attenuation of 10 db is exceeded along a 30-mile path at any fixed azimuth rather more than 8 hours in the season May to September. It serves no useful purpose to impose upon this estimate intervals of time and azimuth for independence of occurrences.

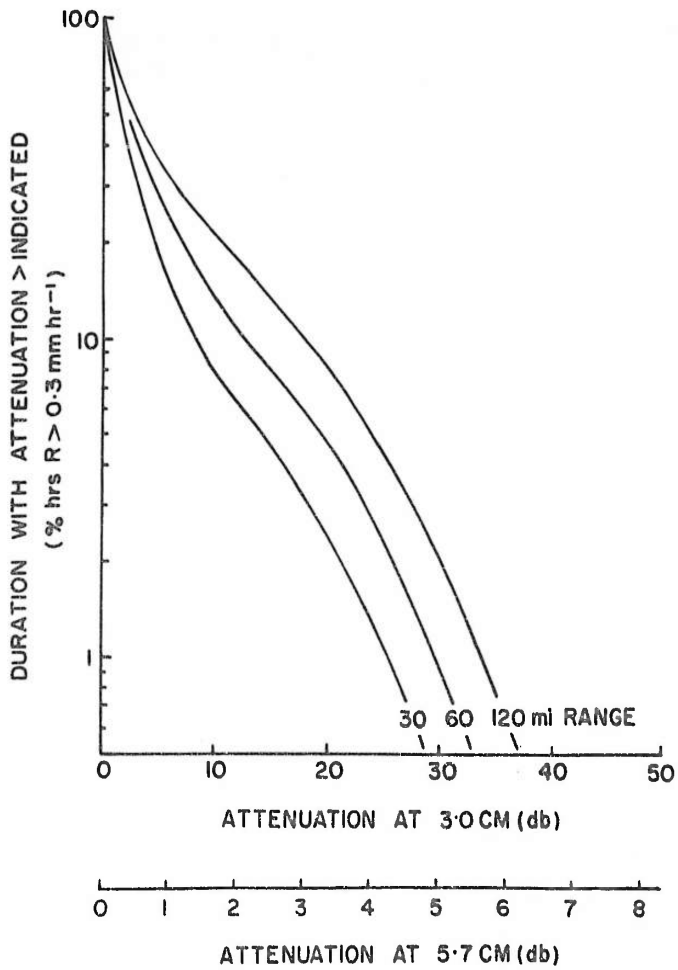
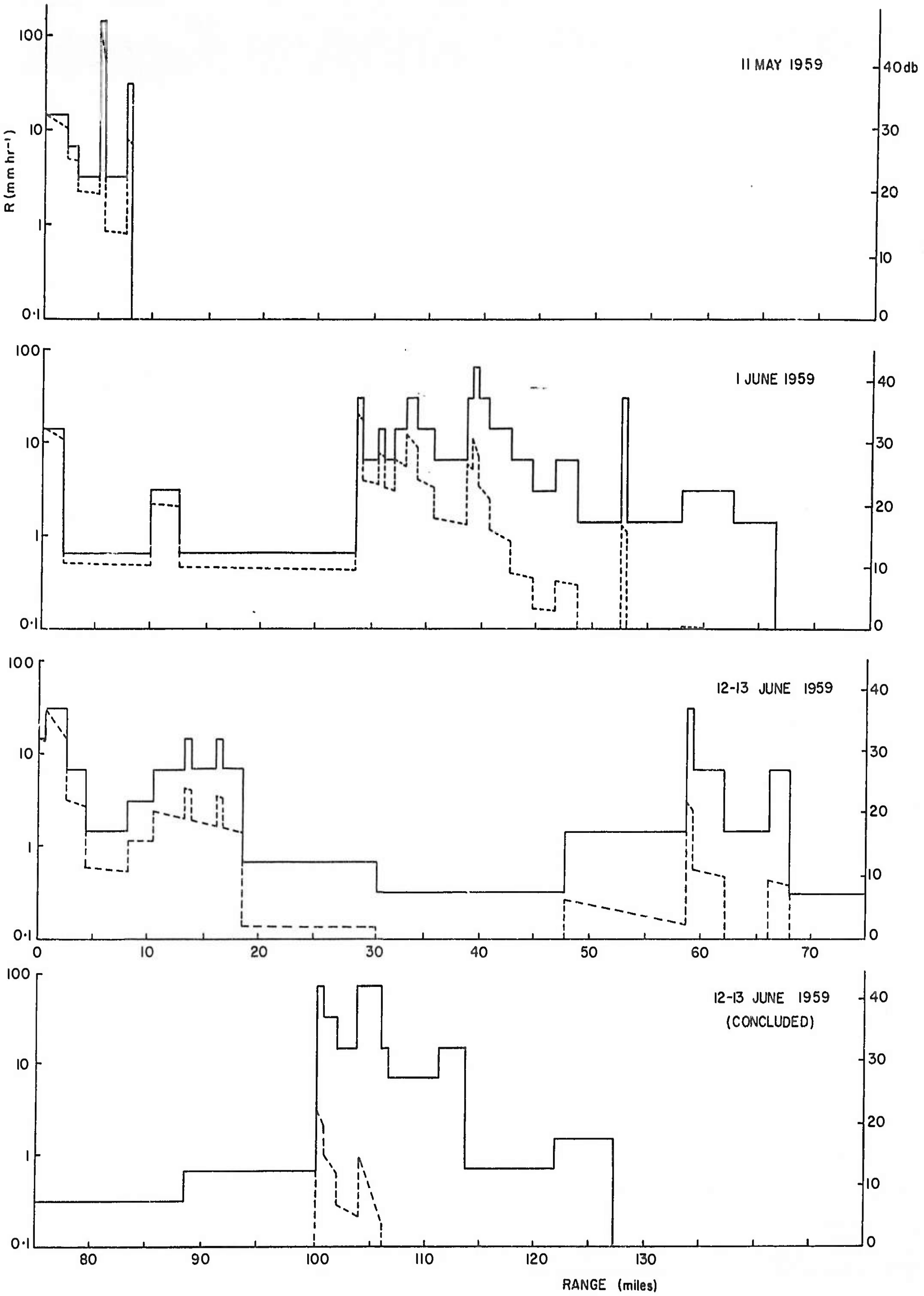


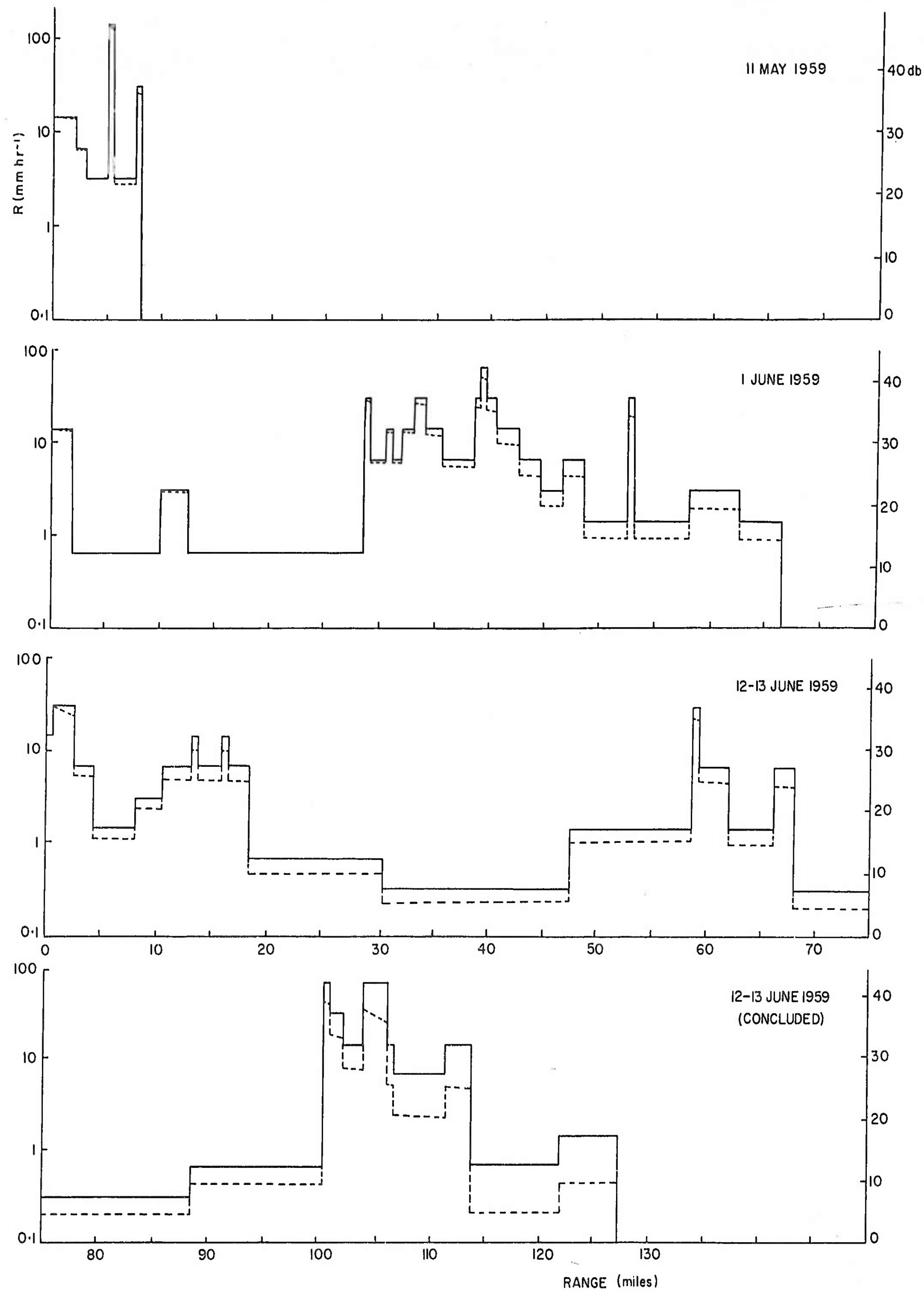
FIG. II.2. Duration for which a given attenuation is exceeded.

APPENDIX III

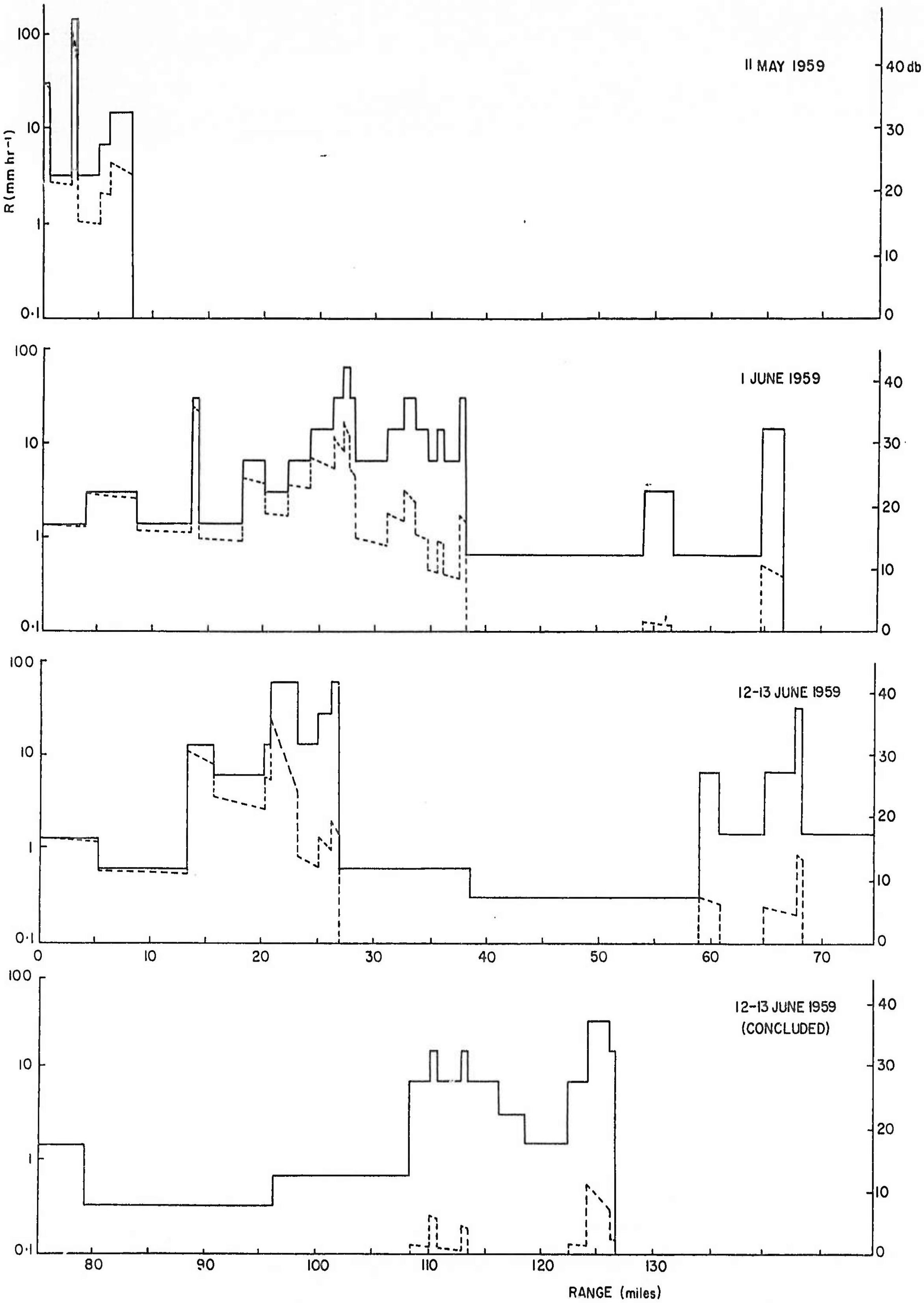
PROFILES OF THE 21 STORMS WITH RATES OF RAINFALL IN EXCESS OF 40 mm hr^{-1}

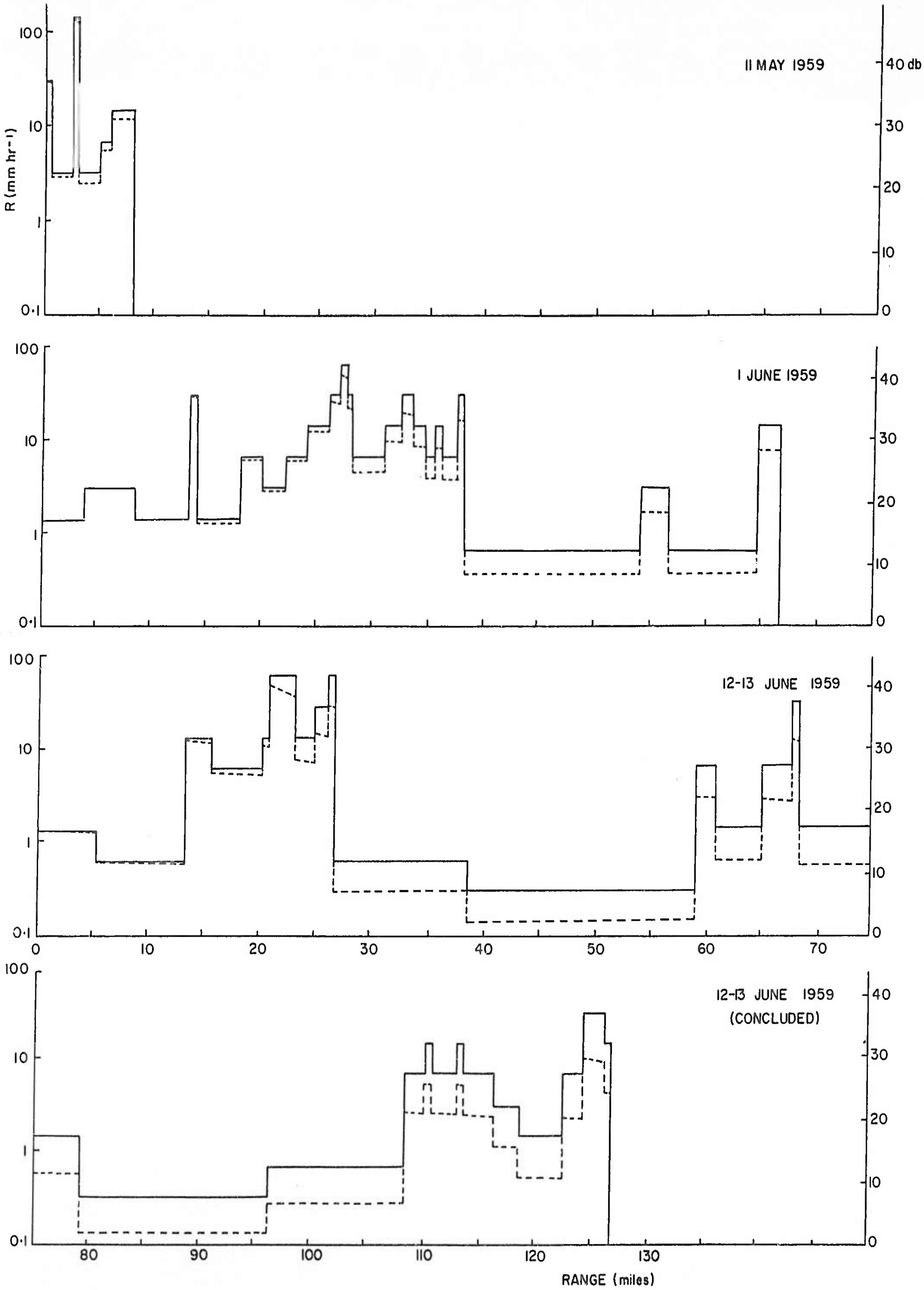
The broken lines are the attenuated profiles, at wavelength 3.0 cm on the left-hand pages, at 5.7 cm on the right-hand pages. Each storm appears twice: First viewed by a radar observing the approaching storm, then, on the next page but one, by a radar observing the same storm departing.

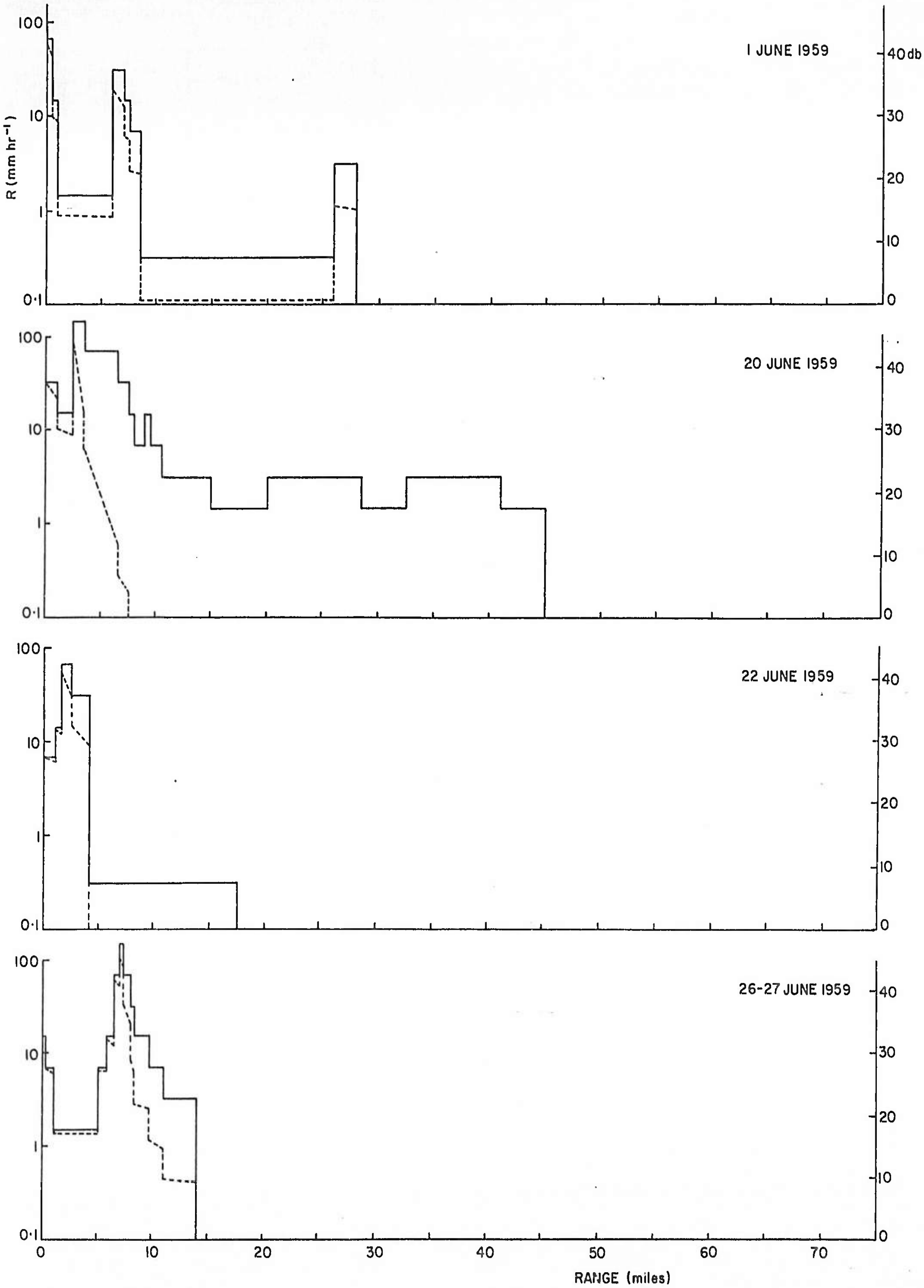

11 MAY 1959
1 JUNE 1959
12-13 JUNE 1959
12-13 JUNE 1959
(CONCLUDED)
R (mm hr⁻¹)
100
10
1
0·1
40db
30
20
10
0
0
10
20
30
40
50
60
70
80
90
100
110
120
130
RANGE (miles)

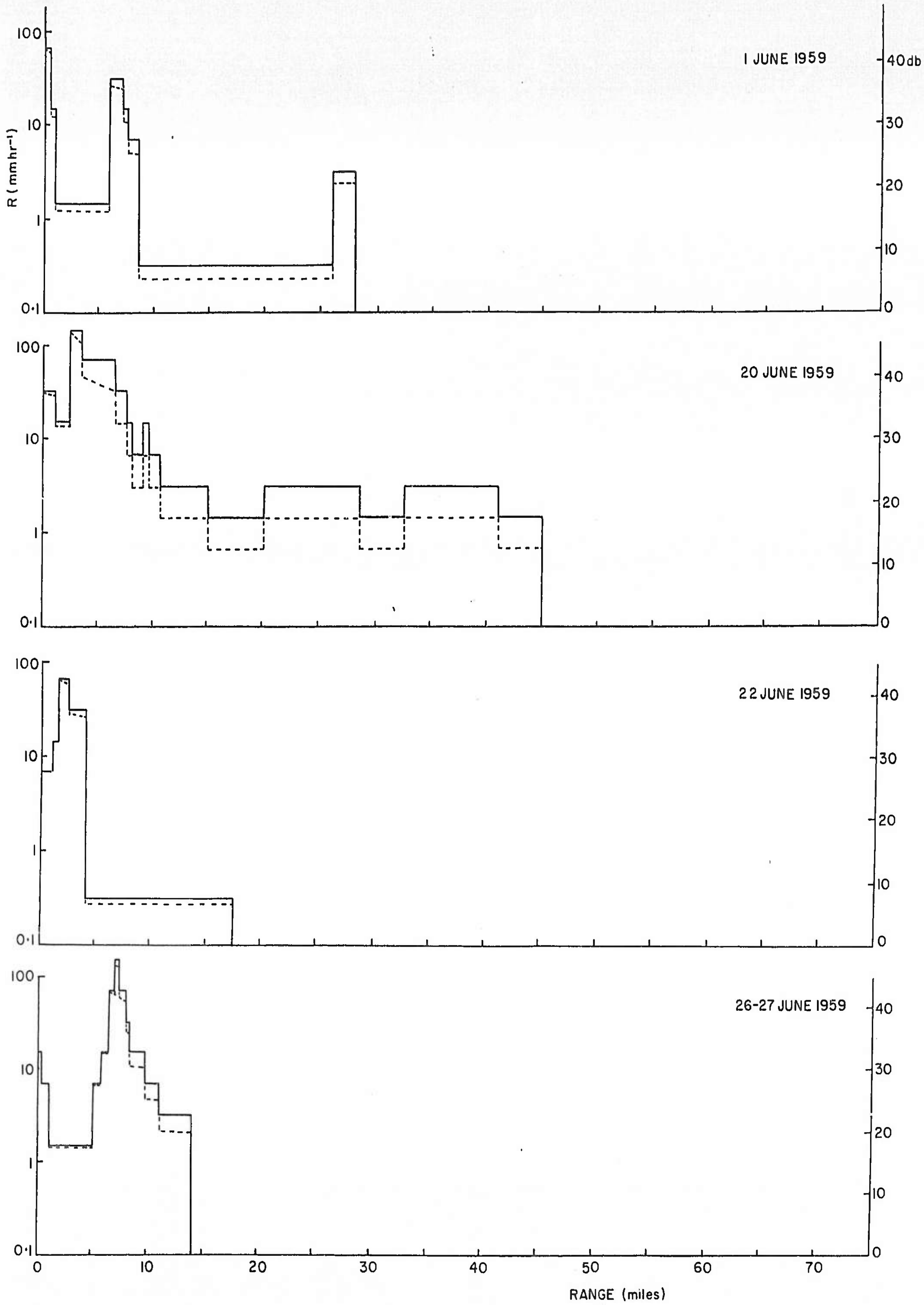


11 MAY 1959
1 JUNE 1959
12-13 JUNE 1959
12-13 JUNE 1959
(CONCLUDED)
R (mm hr⁻¹)
100
10
1
0·1
40 db
30
20
10
0
0
10
20
30
40
50
60
70
80
90
100
110
120
130
RANGE (miles)

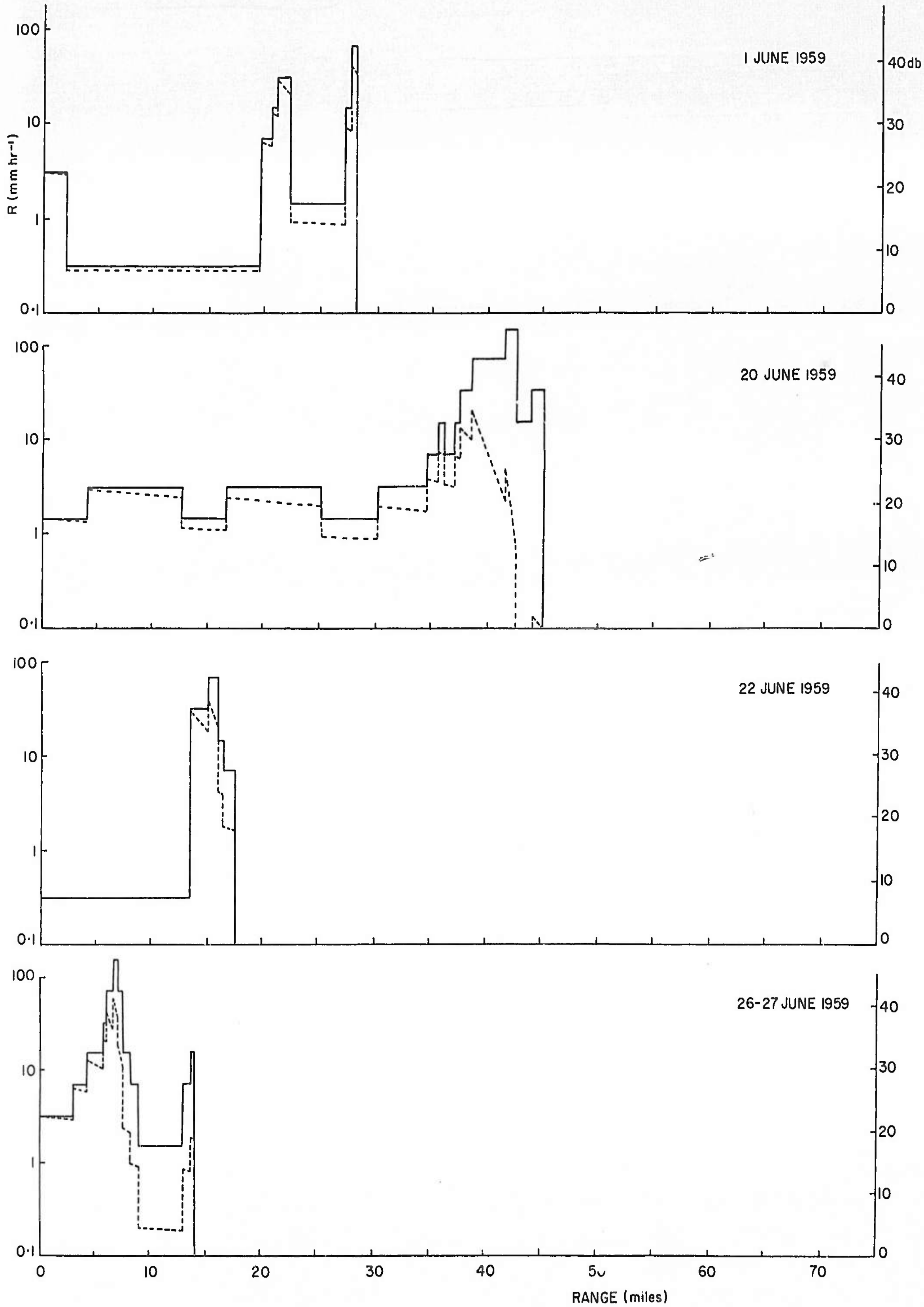

R (mm hr-1)
100
10
1
0·1
11 MAY 1959
40 db
30
20
10
0
1 JUNE 1959
12-13 JUNE 1959
0
10
20
30
40
50
60
70
12-13 JUNE 1959 (CONCLUDED)
80
90
100
110
120
130
RANGE (miles)


11 MAY 1959
1 JUNE 1959
12-13 JUNE 1959
12-13 JUNE 1959
(CONCLUDED)
R (mm hr⁻¹)
40 db
RANGE (miles)

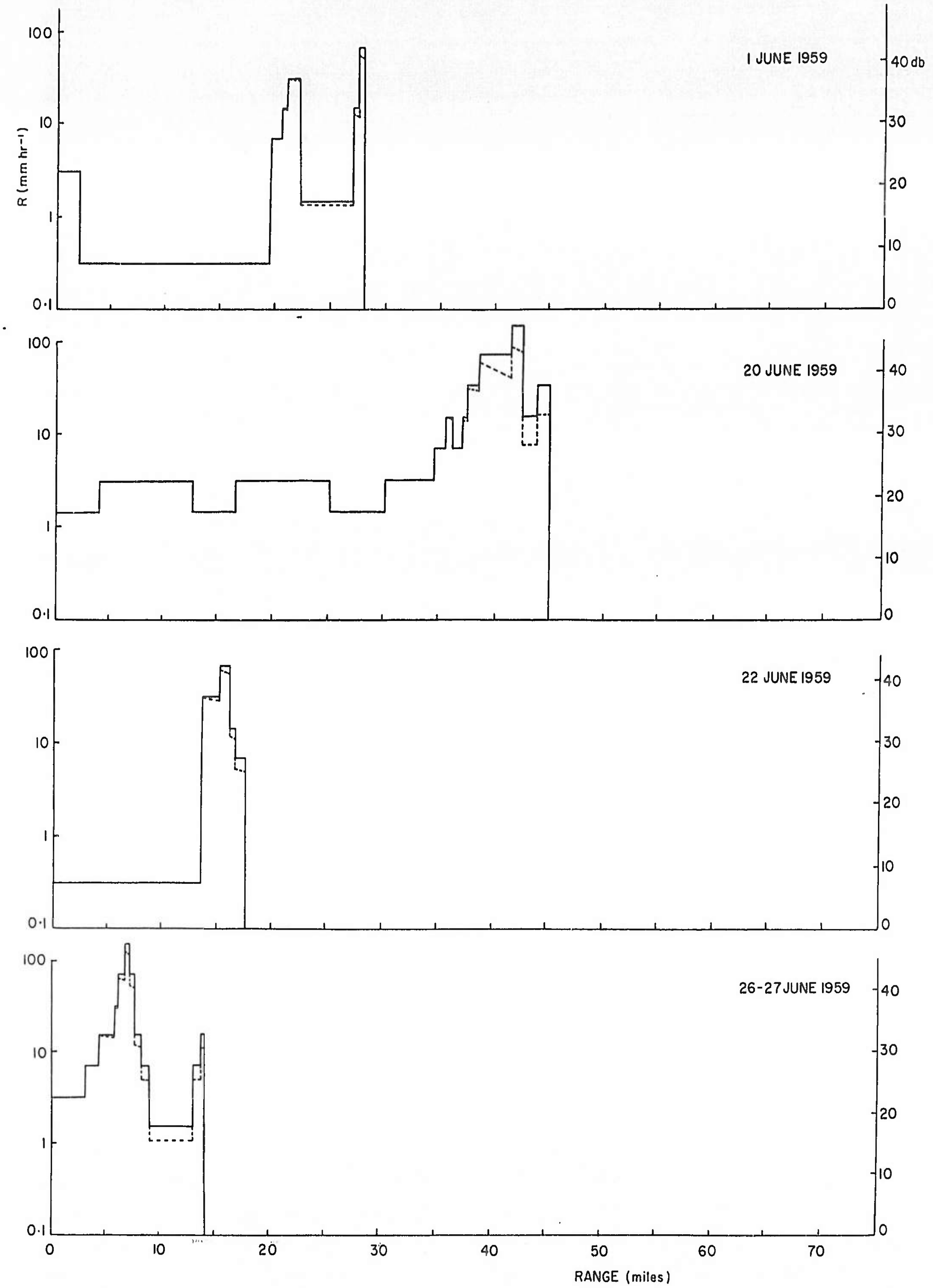

R (mm hr^{-1})
100
10
1
0·1
1 JUNE 1959
40db
30
20
10
0
20 JUNE 1959
22 JUNE 1959
26-27 JUNE 1959
0
10
20
30
40
50
60
70
RANGE (miles)



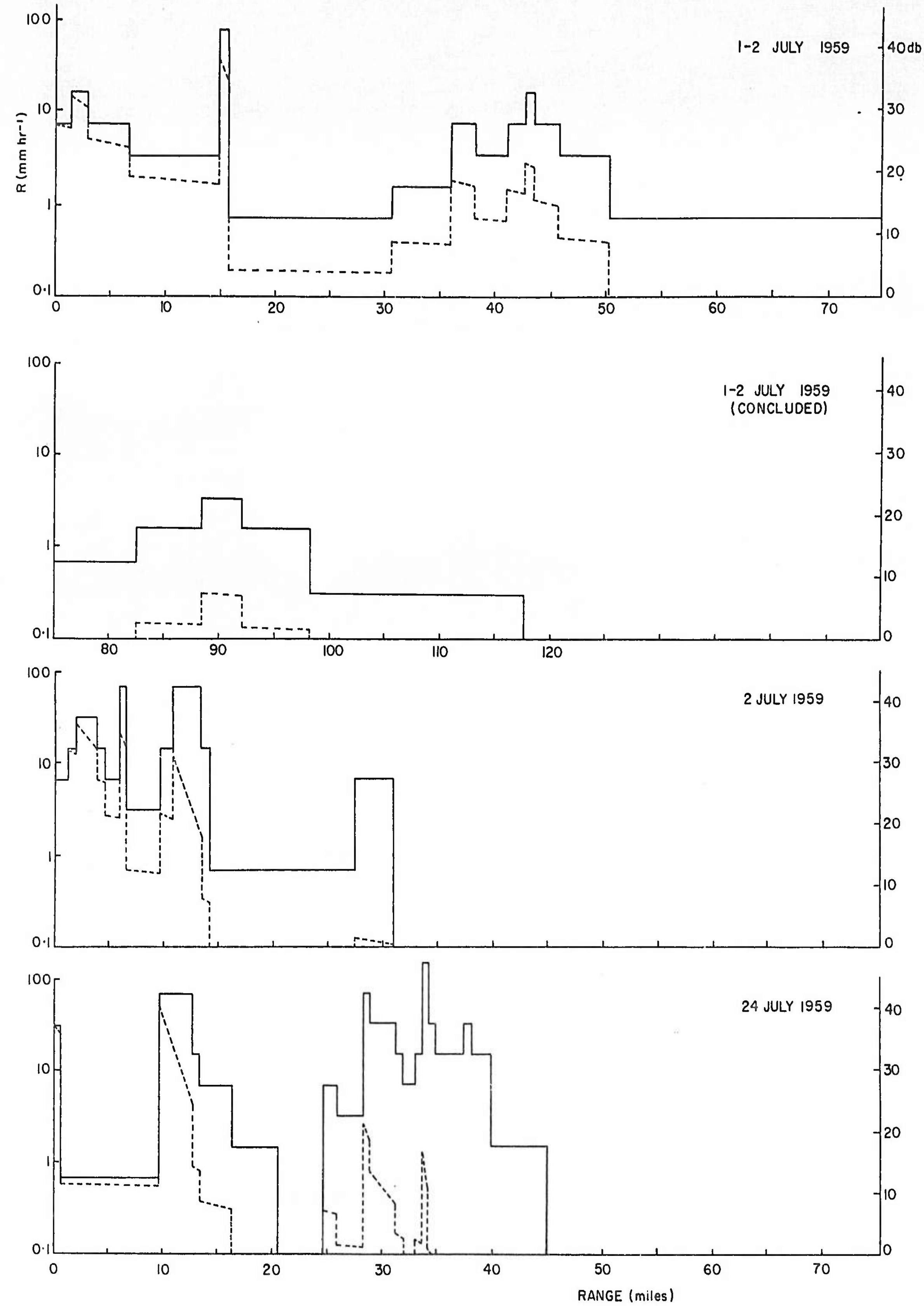
I JUNE 1959
20 JUNE 1959
22 JUNE 1959
26-27 JUNE 1959
R (mmhr⁻¹)
100
10
1
0·1
40db
30
20
10
0
RANGE (miles)
0
10
20
30
40
50
60
70



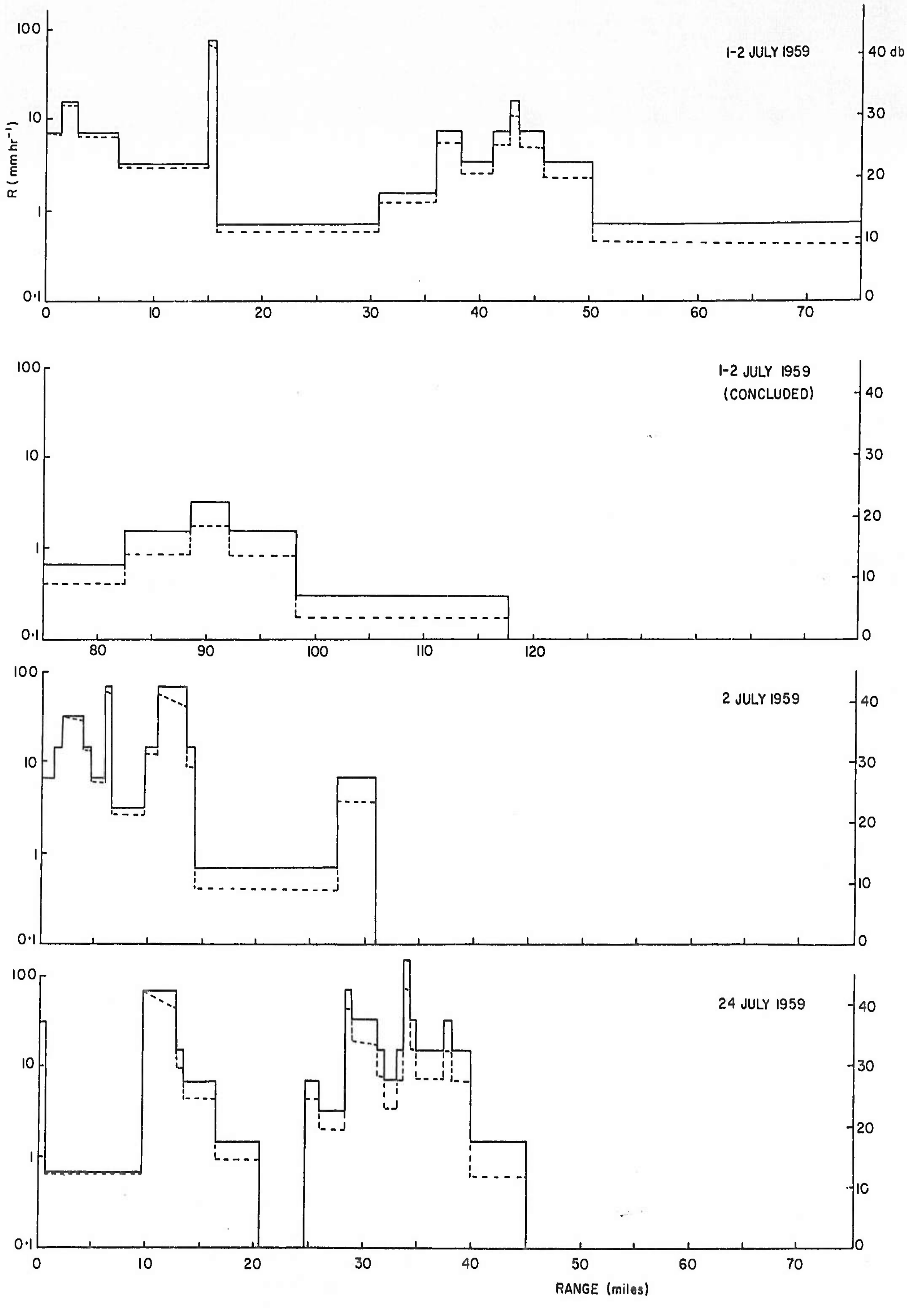
100
10
1
0·1
R (mm hr⁻¹)
1 JUNE 1959
40db
30
20
10
0
20 JUNE 1959
22 JUNE 1959
26-27 JUNE 1959
40
0
10
20
30
40
50
60
70
RANGE (miles)



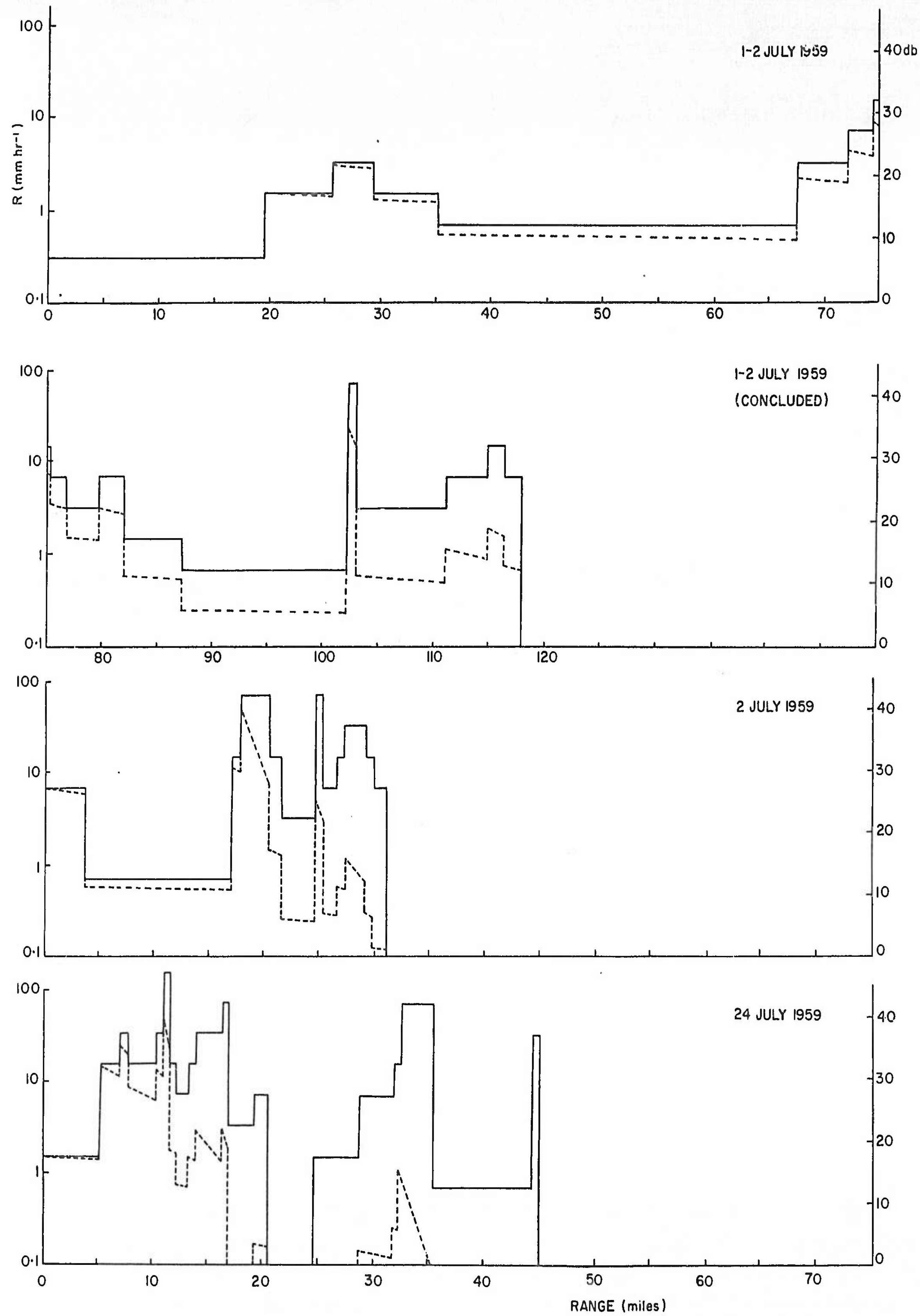
100
10
1
0·1
R (mm hr⁻¹)
1 JUNE 1959
40 db
30
20
10
0
20 JUNE 1959
22 JUNE 1959
26-27 JUNE 1959
0
10
20
30
40
50
60
70
RANGE (miles)

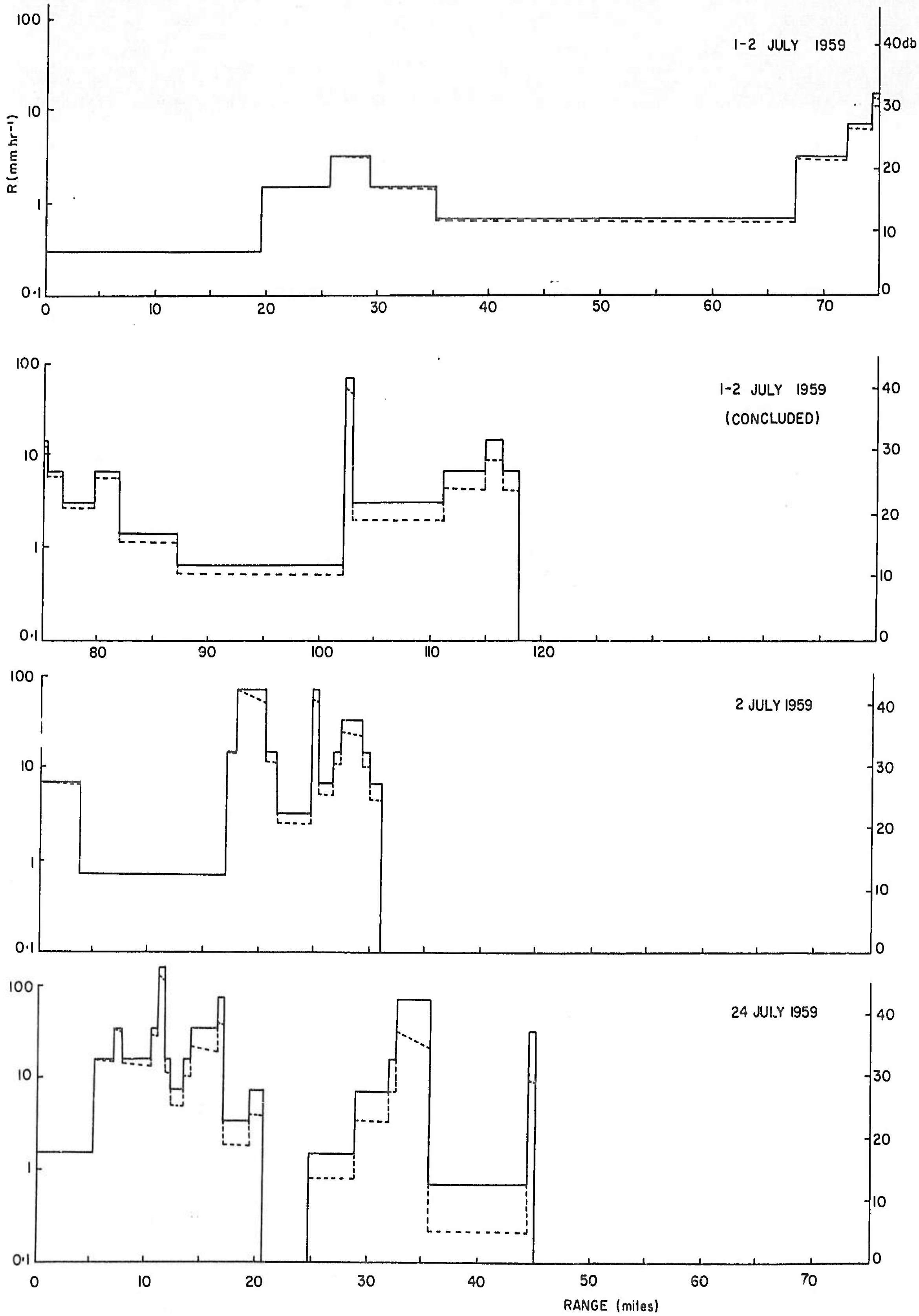


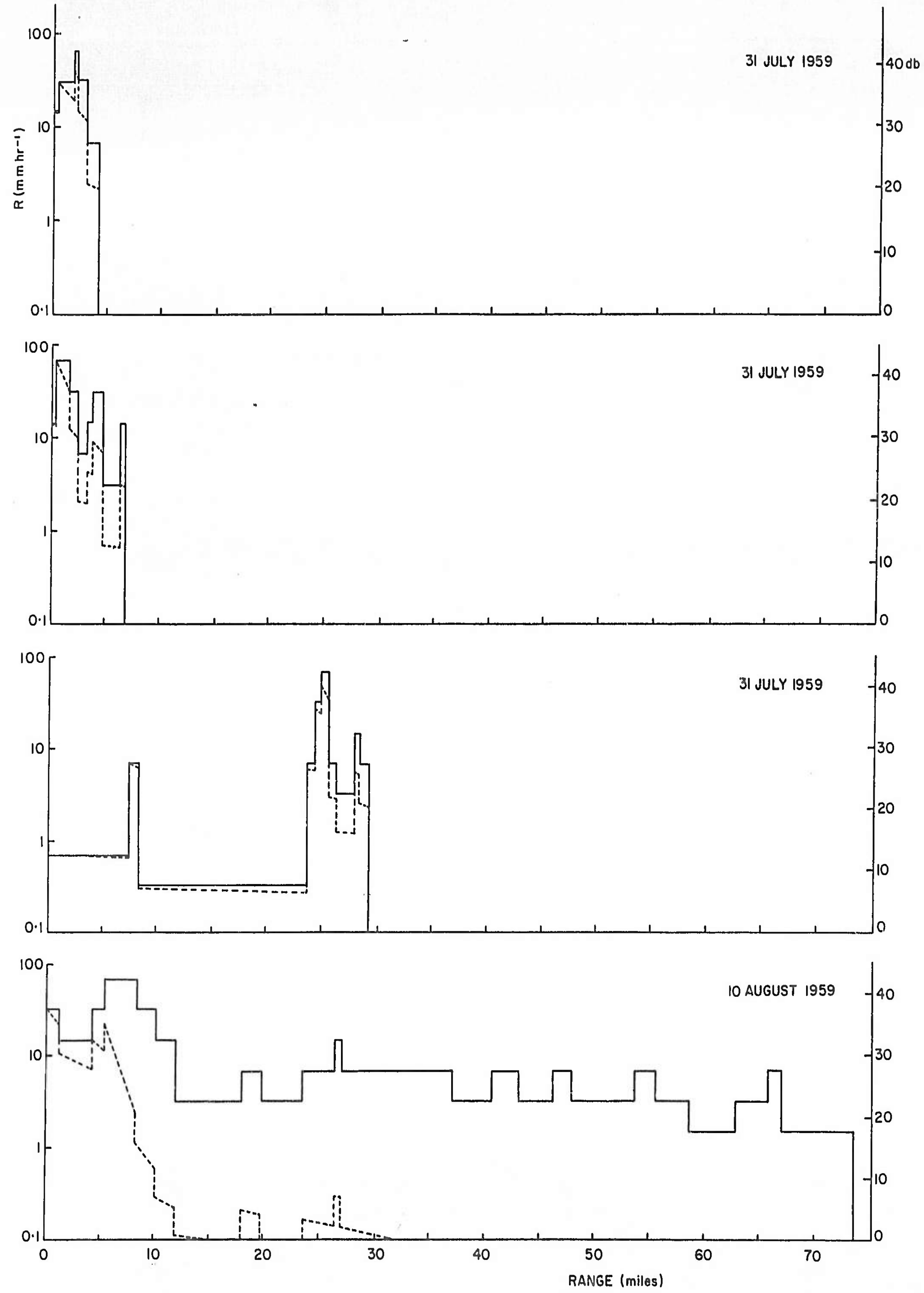
R (mm hr⁻¹)
100
10
1
0·1
40db
30
20
10
0
1-2 JULY 1959
0
10
20
30
40
50
60
70
1-2 JULY 1959
(CONCLUDED)
80
90
100
110
120
2 JULY 1959
24 JULY 1959
RANGE (miles)



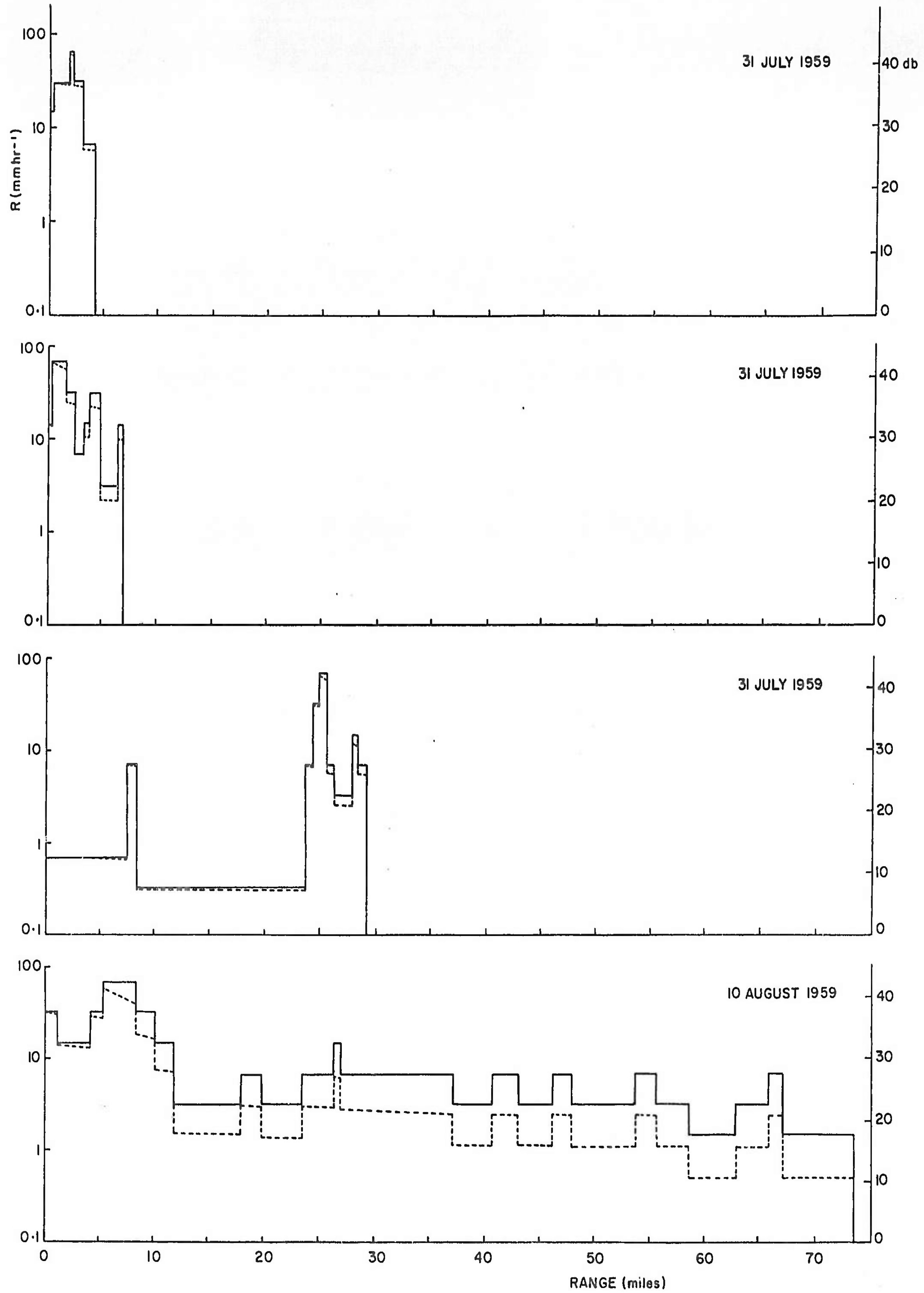
1-2 JULY 1959
R (mm hr⁻¹)
100
10
1
0·1
40 db
30
20
10
0
0
10
20
30
40
50
60
70
1-2 JULY 1959
(CONCLUDED)
80
90
100
110
120
2 JULY 1959
24 JULY 1959
RANGE (miles)

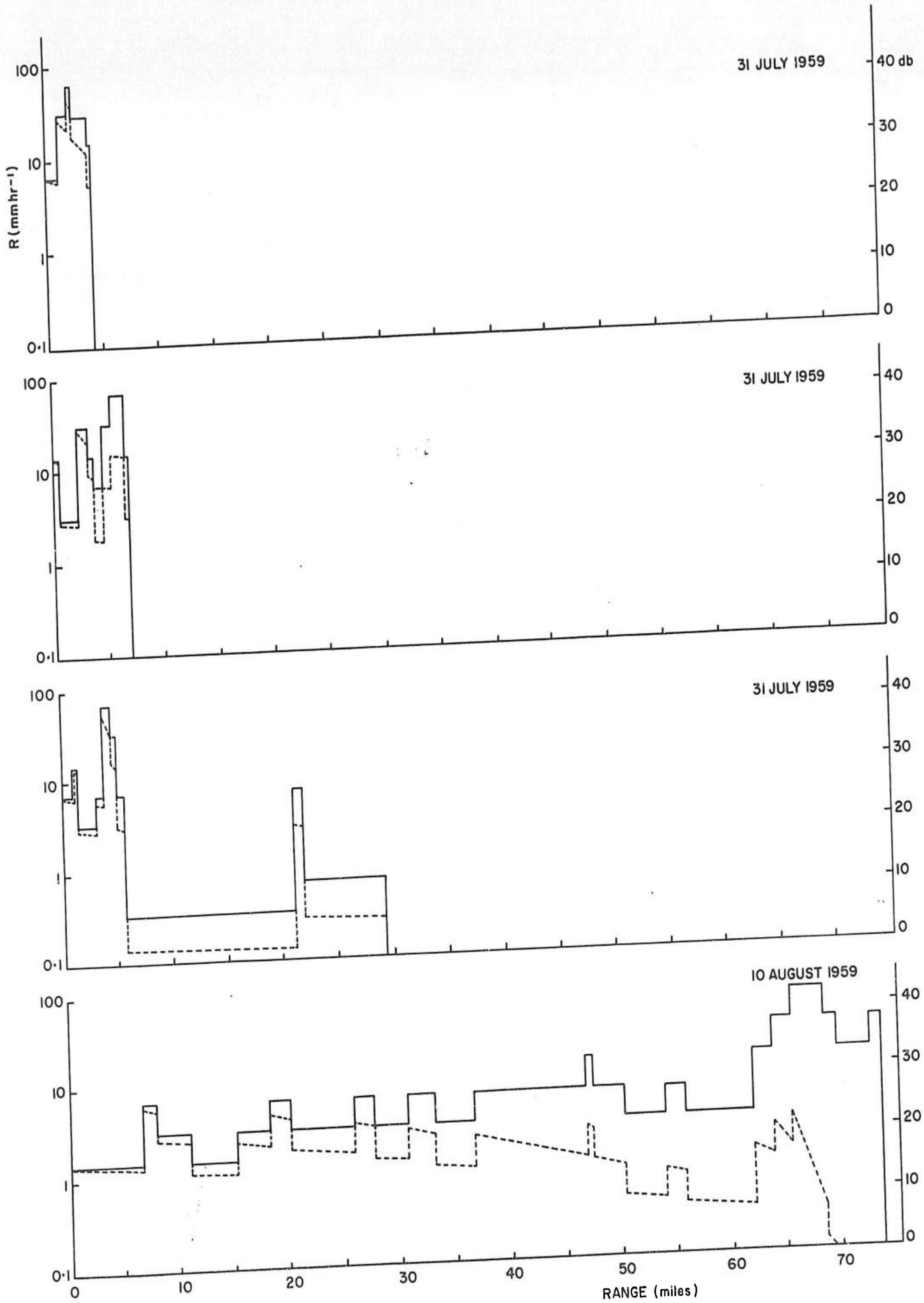

1-2 JULY 1959
R (mm hr⁻¹)
100
10
1
0·1
40db
30
20
10
0
0 10 20 30 40 50 60 70
1-2 JULY 1959
(CONCLUDED)
80 90 100 110 120
2 JULY 1959
24 JULY 1959
RANGE (miles)

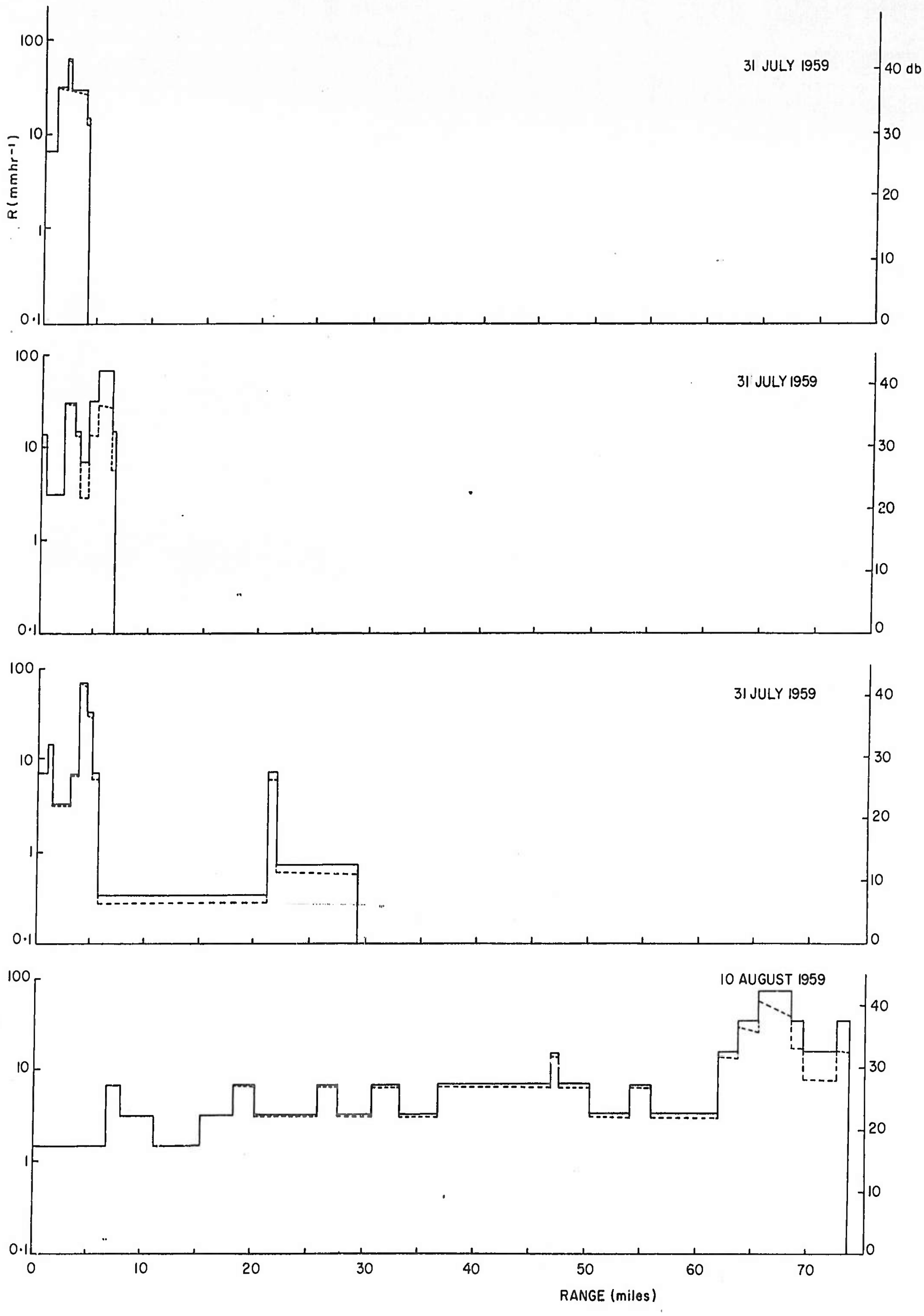

1-2 JULY 1959
1-2 JULY 1959
(CONCLUDED)
2 JULY 1959
24 JULY 1959
R (mm hr⁻¹)
40db
RANGE (miles)

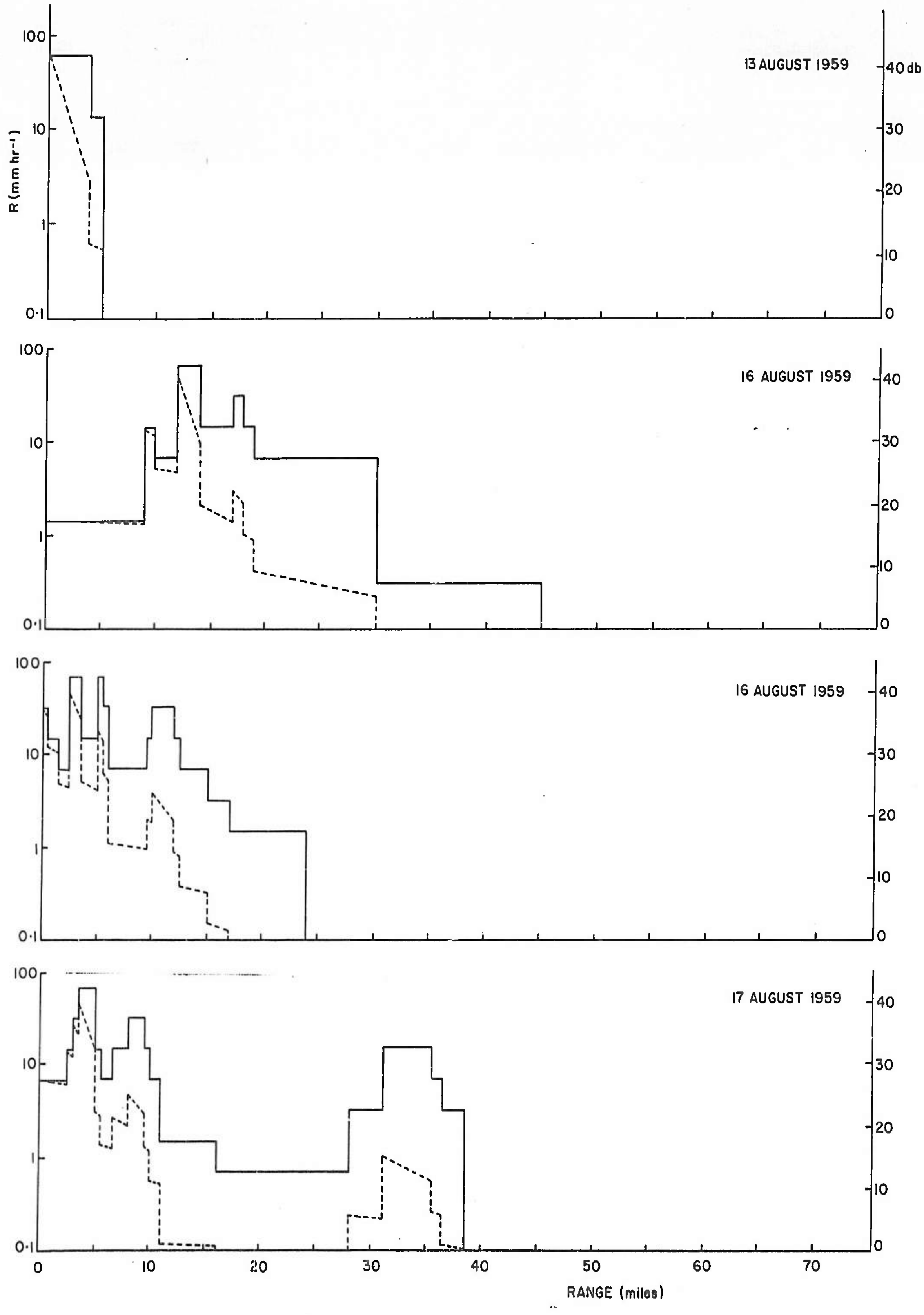


R (mm hr⁻¹)
100
10
1
0·1
31 JULY 1959
40db
30
20
10
0
31 JULY 1959
31 JULY 1959
10 AUGUST 1959
0
10
20
30
40
50
60
70
RANGE (miles)

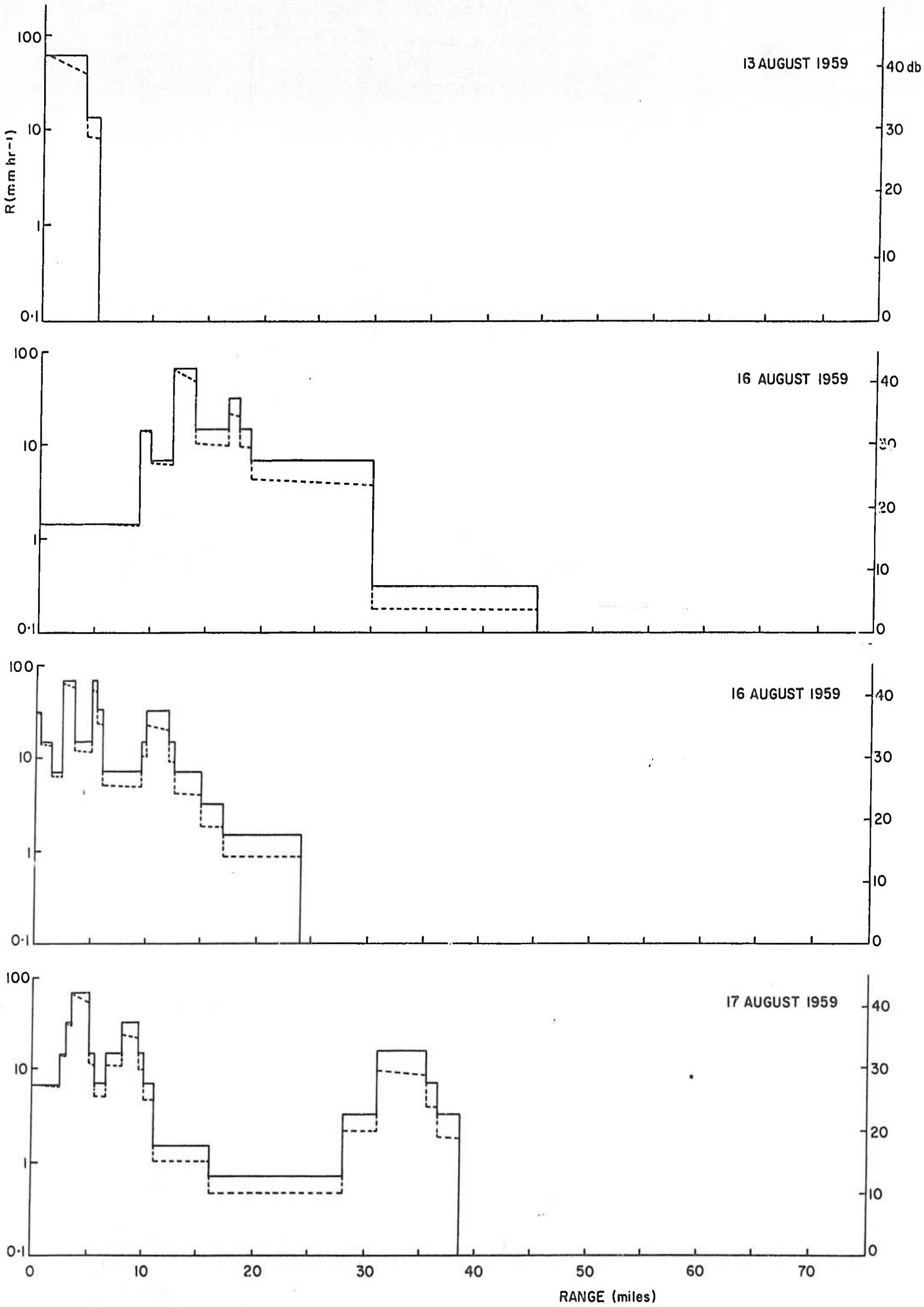

100
10
1
0·1
R (mmhr⁻¹)
31 JULY 1959
40 db
30
20
10
0
31 JULY 1959
31 JULY 1959
10 AUGUST 1959
0
10
20
30
40
50
60
70
RANGE (miles)


31 JULY 1959
40 db
R (mm hr⁻¹)
100
10
1
0·1
30
20
10
0
31 JULY 1959
31 JULY 1959
10 AUGUST 1959
RANGE (miles)
0
10
20
30
40
50
60
70

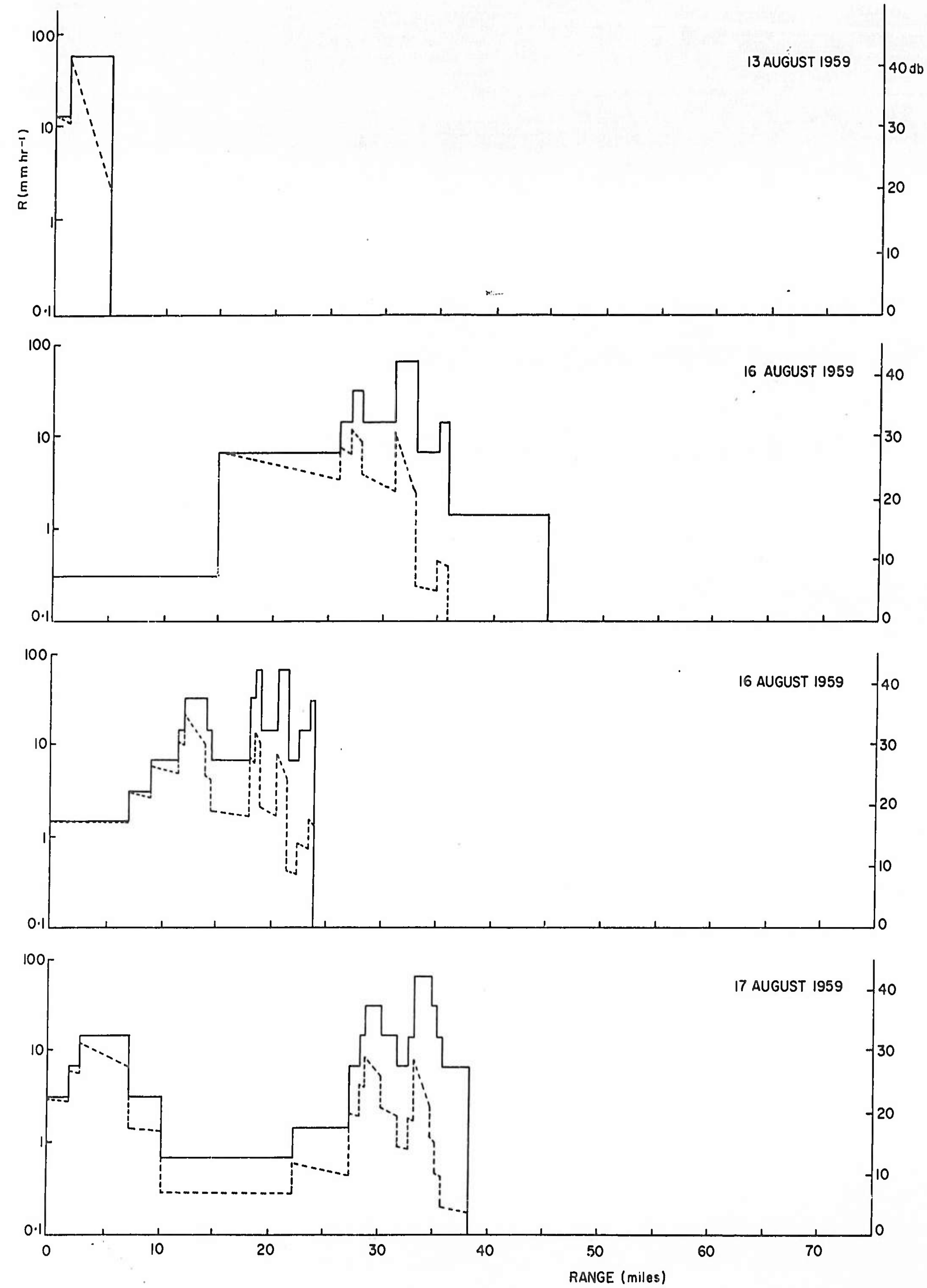

R (mmhr-1)
100
10
1
0·1
31 JULY 1959
40 db
30
20
10
0
31 JULY 1959
31 JULY 1959
10 AUGUST 1959
0
10
20
30
40
50
60
70
RANGE (miles)

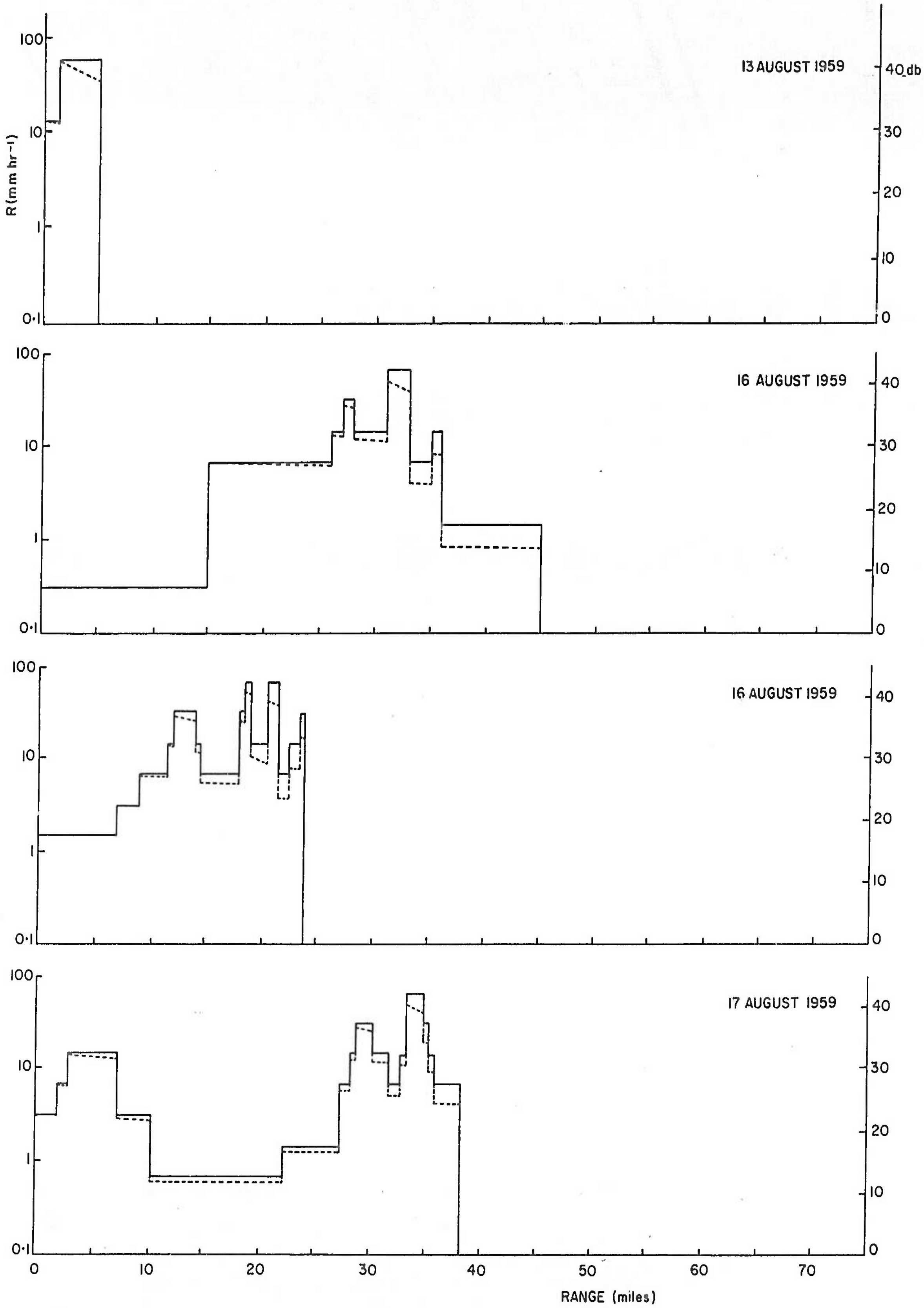


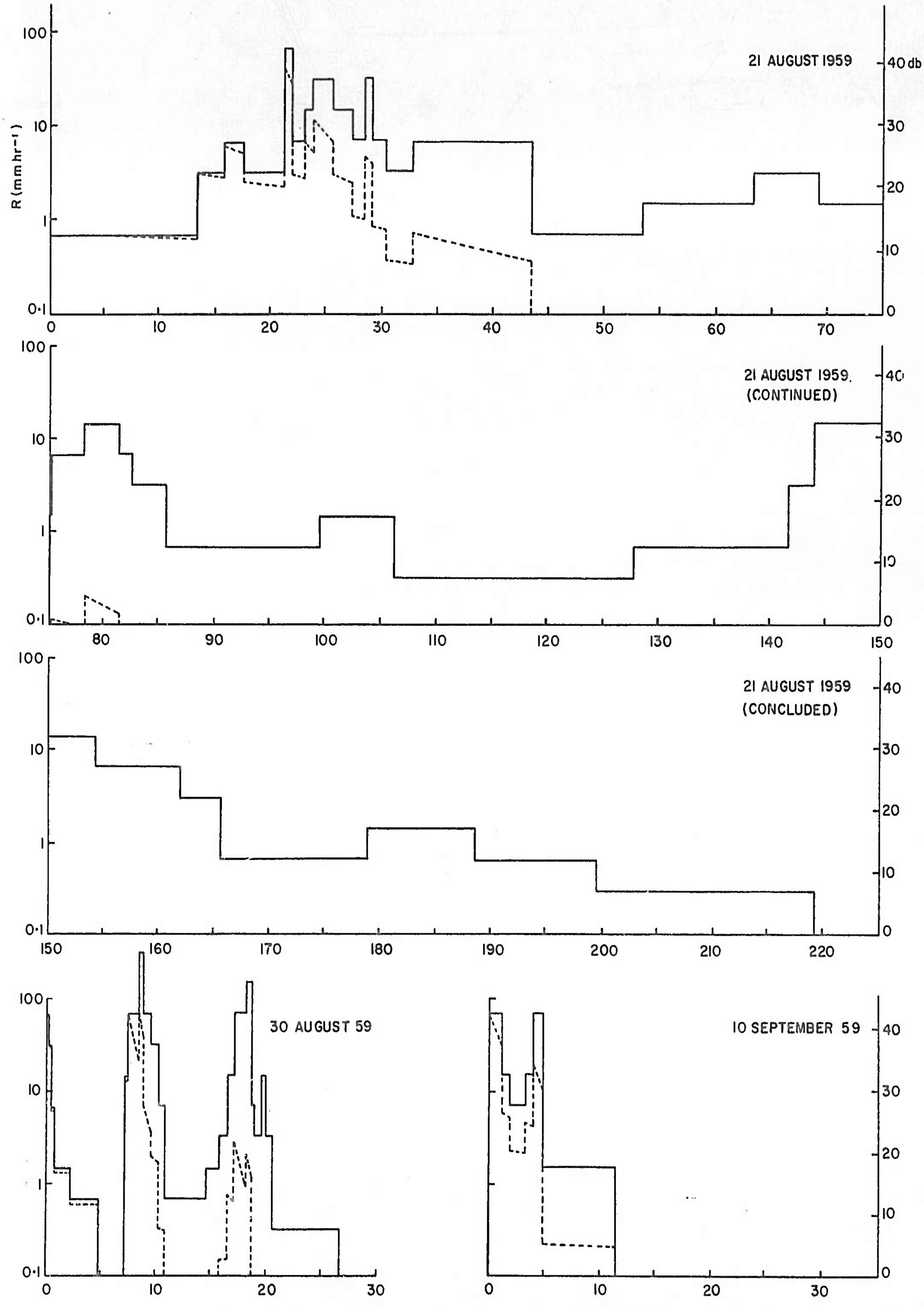
13 AUGUST 1959
16 AUGUST 1959
16 AUGUST 1959
17 AUGUST 1959
R (mm hr⁻¹)
100
10
1
0·1
40 db
30
20
10
0
0
10
20
30
40
50
60
70
RANGE (miles)



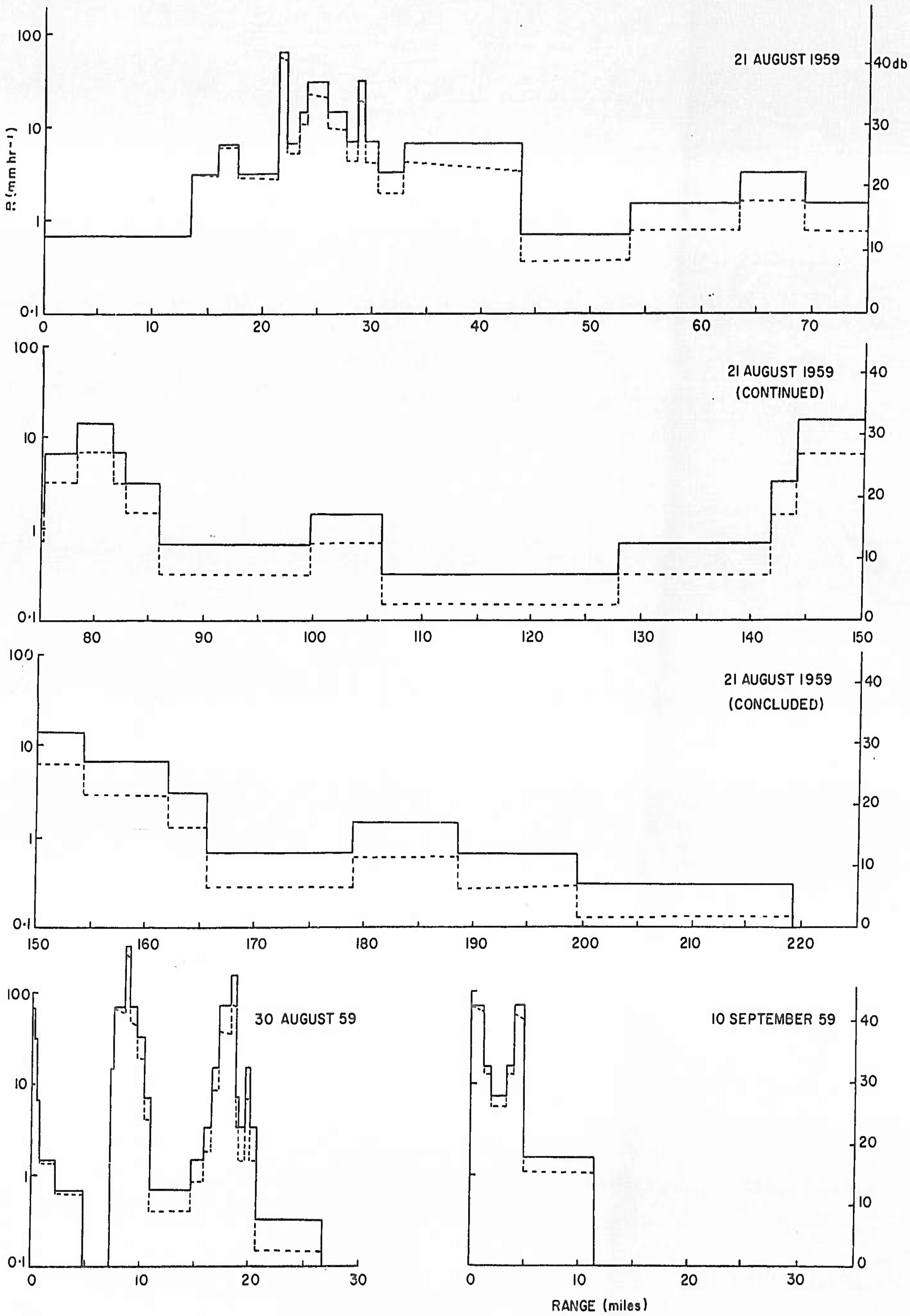
13 AUGUST 1959
16 AUGUST 1959
16 AUGUST 1959
17 AUGUST 1959
R(mm hr⁻¹)
100
10
1
0·1
40 db
30
20
10
0
0
10
20
30
40
50
60
70
RANGE (miles)

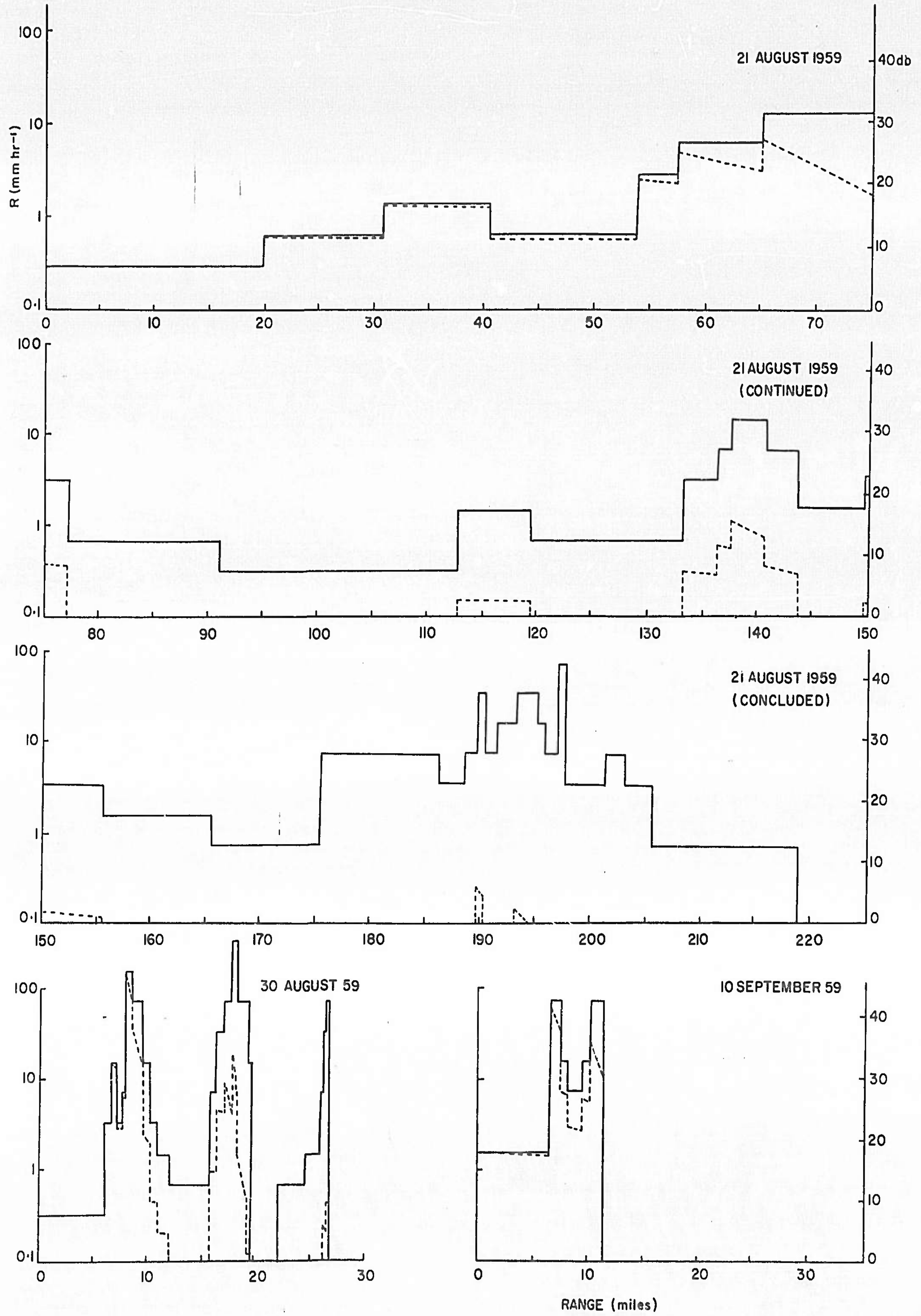

100
10
1
0·1
R (mm hr⁻¹)
13 AUGUST 1959
40 db
30
20
10
0
16 AUGUST 1959
16 AUGUST 1959
17 AUGUST 1959
0
10
20
30
40
50
60
70
RANGE (miles)

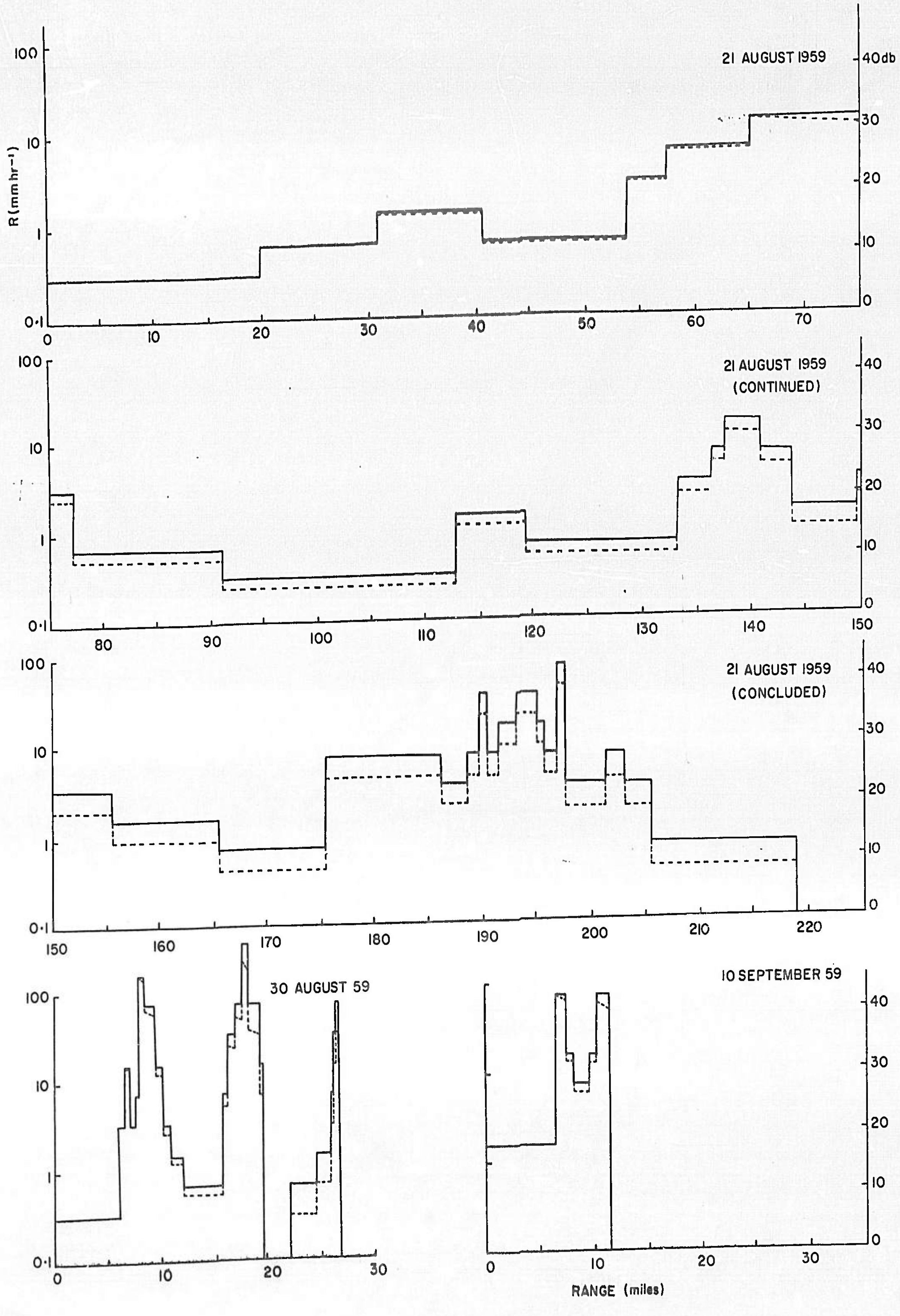

13 AUGUST 1959
16 AUGUST 1959
16 AUGUST 1959
17 AUGUST 1959
R (mm hr⁻¹)
40 db
RANGE (miles)



R (mm hr⁻¹)
100
10
1
0·1
21 AUGUST 1959
40 db
30
20
10
0
0
10
20
30
40
50
60
70
21 AUGUST 1959
(CONTINUED)
80
90
100
110
120
130
140
150
21 AUGUST 1959
(CONCLUDED)
160
170
180
190
200
210
220
30 AUGUST 59
10 SEPTEMBER 59
RANGE (miles)


R (mm hr⁻¹)
21 AUGUST 1959
21 AUGUST 1959 (CONTINUED)
21 AUGUST 1959 (CONCLUDED)
30 AUGUST 59
10 SEPTEMBER 59
RANGE (miles)


21 AUGUST 1959
21 AUGUST 1959 (CONTINUED)
21 AUGUST 1959 (CONCLUDED)
30 AUGUST 59
10 SEPTEMBER 59
R (mm hr⁻¹)
40db
RANGE (miles)


21 AUGUST 1959
R (mm hr^{-1})
40db
21 AUGUST 1959
(CONTINUED)
21 AUGUST 1959
(CONCLUDED)
30 AUGUST 59
10 SEPTEMBER 59
RANGE (miles)